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A Sceptical Note on the So-Called Technical Relations' in Agriculture

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1. The Theme:

When certain notions in economics get sanctified by repeated usage scepticism about their general validity as well as applicability in particular cases gets thwarted: professional practitioners and the followers, gaining rapidly in number, understand what it all means and communicate in an exclusive parlance accepted without questioning. Often some notions, conceived of as an integral part of a comprehensive rigorous theoretical structure are divested of their original context, put to empirical tests in isolation and endowed with an interpretation that had validity only in the original theoretical frame.

One such notion which due to its apparent theoretical versatility and empirical content has gained much favour with research scholars in Agricultural Economics is that of the production function, as an economists' tool for summarising technological information and deriving 'technical relations' with important economic-theoretic implications. The uses that the function has been put to are many. At the macro level, to test for the existence of surplus labour by comparing the marginal value product of labour estimated from a fitted aggregate function to the prevailing wage rate; to test the hypothesis of rational resource allocation at the farm level by testing for the equality of the estimated marginal value product of the same input in different uses and the equality among the ratios of marginal value products of different inputs to their respective prices; to relate productivities of factors to the size of holding etc. In this note we wish to raise some simple questions - some of them old but which nevertheless appeared to have been ignored and hence bear repetition - concerning the use of the production function to describe technical relations and to derive economic theoretical propositions therefrom. We have focused attention particularly on the issue of 'returns to a factor' and 'returns to scale' that have frequently appeared in a number of empirical studies on Indian agriculture and have provided the basis for inferring theoretical conclusions.

2. Production function and its different connotations:

Production function as a tool for depicting possibilities of transforming inputs into outputs has, in fact, three distinct connotations. First it could be viewed as a set of technical processes - a 'book of blue-prints' - which the existing state of engineering and technical knowledge offers as feasible. As such these are physico-technical relations and to the extent stated in physical units of various inputs and products are independent of valuation. Secondly, there are statistically fitted production functions, which, adopting certain statistical procedures, estimate the relation between inputs and outputs in terms of some formally defined relation, commonly in the form of a continuous function. The relation gives an idea about the statistical association that may be inferred to underlie a set of empirical observations on inputs and outputs.

The production function in economic theory should be strictly speaking distinguished from both the above notions. As an economic concept it is a logical construct giving the maximum output Y, which can be produced by a given set of inputs $(x_1, x_2, \cdots x_n)$; this catalogue of possibilities defined by $y = \phi(x_1, x_2 \ldots x_n)$ is defined over the region for which $y > 0$ and ϕ_i, the first partial derivative with respect to an input i, is nonnegative (i.e. $\phi_i \geq 0$). These are considered to be given independently of prices and constitute only the 'efficient' combinations (i.e. only those combinations appear on the production function which, to produce the same output, involve no more of any one input without using less of any other(s) when compared to the rest of the combinations.) Ideally speaking, like the engineering blueprints, these too must be defined without prior knowledge of prices. However, in so far as inputs and/or outputs are value aggregates and not expressed in technical units, the function ceases to have that independence of prices. Also, while the engineering processes give a listing of inputs and outputs, the economic production function in its aggregate form, as used especially in the macro theories of growth and distribution, attempts to reduce all input requirements to 'primary' factors, commonly 'labour' and 'capital'. This reduced form of the production function relates not output directly to the 'primary factors', eliminating the intermediate inputs in the process. When 'capital' appears thus as one of the 'primary' factor, it is stated in terms of value and this, as we shall see, raises certain severe problems for the marginal productivity explanation of distribution. Also to be used in the supply and demand based explanations of value and distribution, the economic production function must satisfy certain properties e.g. the marginal product of any single input must be nonnegative and decline with increasing use of the input. (Also the second order cross partial derivatives must be positive, i.e. the marginal product of an input must be higher for higher levels of other inputs).

As distinct from the statistical production function, the economic production function gives a set of ex ante alternatives from which the producer chooses the optimum technique, given his objective function and the prices of inputs and outputs. The statistical production function, on the other hand, is based on ex post observations, i.e. on chosen techniques already in practice. They are not based, that is, either on experimental data or on randomly generated data. The prevelent techniques in use are themselves results of production decisions. This is important to be borne in mind. Further the statistical production function fitted to such observations is different from the economic production function in another important way. It is based on certain 'average' relation between input and output, the 'average being defined by the estimation procedure. e.g. under regressional estimates the production function is obtained by the procedure of 'least squares Thus the statistical function does not give the set of the 'most-efficient' techniques as does the theoretical production function.[1] In fitting the function statistically it is presumed that certain variations in output occur due to errors while inputs are accurately measured.

Quite often it would seem that these three distinct connotations of technical relations are confused so that sometime the economic production function stated in terms of inputs, some of which are value-aggregates, is treated as though it constituted purely physical relations of an engineering sort, although no engineering possibilities as are depicted by the economic function, e.g. of continuous inputs substitution, may realistically obtain. The statistically fitted function is sometimes interpreted as though it were the theoretical function and certain properties stipulated on the latter are ascribed to the former although it may be evident that the assumptions under which the theoretical relations are obtained are violated in the observed situation. This questionable traversing from one notion to another creates many anomalies in hypothesis building, testing and interpreting the results. We shall be concerned here with a few such cases pertaining to 'returns to a factor' and 'returns to scale' interpreted as 'technical relations' and certain economic theoretical results based upon their use in agricultural studies.

3. 'Return to scale': diverse uses of the term.

The term 'returns to scale' itself is used with diverse meanings. It was originally conceptualised (particularly in the 'diminishing returns' form) in the context of an expansion in output calling upon the use of increasing doses of variable factors applied to a fixed factor. It was presumed that changes in costs could be conceived as functionally related to changes in the scale of output. In this context increasing returns was taken to be synonymous with decreasing average costs and diminishing returns with increasing average costs. The laws of returns thus conceived provided the basis for the supply curve of individual commodities and for the derived demand curve for 'factors' of production. This construction of the supply curve formed an important part of the traditional explanation of value and distribution. There is another, more specific, sense in which the concept 'returns to scale' is used; viz., to characterise the extent of changes in output following upon a proportional expansion of all inputs. Constant returns to scale obtain when expansion in output occurs by a proportionality factor equivalent to the scalar change in the vector of inputs (in other words, with an equal proportionate change in all inputs). When the change in output following upon a uniform proportionate change in inputs is less than (or more than) proportionate, it is decreasing (increasing) returns to scale. This notion has been brought into discussions of technological economies of scale. Thirdly, in some recent discussions on Indian Agriculture, returns to scale has been used in the context of changes in productivity of individual inputs (particularly, land) with changes in the scale of production, the latter usually denoted by the size of holding. From the point of view of theory this particular use of the term is rather loose. We shall take up these in seque.

I.1. Returns to a variable factor

The notion of returns to scale, especially in the form of diminishing returns to scale was traditionally associated with the case of the application of a variable factor to another (or a set of) constant factor(s) held at a fixed level. The notion originated in the classical case of diminishing returns in cultivation where, as output was increased through extensive cultivation (by moving on to inferior qualities of land') or through intensive cultivation (by applying larger doses of investment, 'labour and capital', to the same piece of land), the returns to the successive units of investment was presumed to decline. This classic case was invoked in relation to the theory of distribution, to explain rents where 'land' was taken as 'fixed' for the entire community. The relevant producing unit was not the individual producer but the community at large. Land as a non-reproducible asset was considered physically limited to the available land-mass of the community.[2]

Generalization in terms of diminishing returns to a factor:

In the subsequent supply and demand theories of value and distribution this notion however was generalised to all cases of production (including industry) all units of production (including the individual producer) and to all factors of production. The notion of relative scarcity and hence fixity in the sense of availability in 'given' amounts of factors, was made applicable to 'capital' as well as land. The laws of increasing and diminishing returns (in other words, decreasing and increasing costs) provided the basis for the supply curve. Further, a generalised theory of factor-substitution was built up whereby factor-proportions responded to factor prices and a supply and demand based theory of distribution was constructed.

The idea of substitution and of 'diminishing returns to a factor' was extended also in another direction. The earlier case of diminishing returns occurred in the case of agriculture, when, as a result of increasing population, cultivation was extended and therefore the diminishing returns to the variable factor was clearly due to increase in output, in response not so much to change in prices as in population. The theory of factor-proportions, based on substitution of one factor for another, was extended to the case of constant output; wherever a factor, say 'capital', was applied more intensively to labour than before, maintaining the same output level (i.e. along the same isoquant) it was believed that the return to the factor 'capital' (or, the marginal productivity of capital) would decline. That is, the earlier theory of returns to scale was now generalised in terms of returns to a factor varying with changes in factor-proportions. Thus the formulation acquired a general form: whenever more units of a factor are combined with the same (fixed) units of another to produce more output, or with appropriate reduction units of another so as to maintain output, its marginal productivity declined. In the former case the additional product due to the additional units of the factor declines while in the latter, it appeared in the form of increasing marginal rate of substitution, i.e., increasingly more units of the factor would be required to maintain the same level of output as one factor is progressively substituted for the other, reflected in the assumption of strict convexity of a typical isoquant.

The notion of marginal productivity appears at both the macro level and micro level, i.e., in the theory of distribution where the 'factor rewards' (wages, rents, profits per unit of the factor) are sought to be explained by the marginal value product, measuring the contribution to output of the factor concerned and to explain resource utilisation by micro-producing units. In the latter, factor as well as product prices are taken provisionally as 'given' by the market and remain independent of the production and investment decisions of the particular producer concerned. The efficiency of resource use by the producer is discussed under the usual premise of profit maximizing behaviour and conditions derived for the optimal degree of resource use; the condition being that the producer, given possibilities of continuous substitution among factors, extends the use of any particular input to the point where its marginal value product is equal to its price. In both these uses; i.e. in explaining factor rewards and resource use, there arise a number of logical and other difficulties, although of some what different order in the two cases. We shall discuss some of them in the following.

I.2. Returns to a variable factor: is it a technical relation?

The notion of returns to a variable factor is sometimes talked of in a purely technical and physical sense i.e. independent of valuation, in a rigid setting of a simulated experiment. The underlying conception suggested appears to be that of an engineering one. A variable input, homogeneous in quality, is applied successively in uniform doses to an input or a set of inputs held at a fixed level to produce a single output (or a bundle of outputs of fixed composition), again uniform in quality. It is presumed that as the 'doses' of the variable input increase, sooner or later, the additional product due to the additional dose declines, yielding the negatively inclined marginal productivity schedule. This conception ideally speaking, is free of valuation, based as it is on purely physical magnitudes. The marginal productivity schedule is implicitly used as such a purely technical relation even when the 'variable input'/fixed factor/ output is a value-sum of heterogeneous elements. This raises problems for the macro theory of distribution as well as micro level problem of resource utilisation.

Quite often, except under highly artificial conditions of simulated experiments these conditions defining the marginal productivity of a factor may not be satisfied empirically. It may not be technologically possible to isolate one input and combine increasing amounts of it with others held at a fixed level, to produce the <u>same type of output</u>. Moreover such <u>continuous</u> variations of an isolated input may not be technologically permissible: the extreme example is that of fixed proportions, where the 'marginal product' of an incremental unit of any single input could be zero.

In fact such an incremental dose of the input may alter the nature of the product or destroy the entire product as may happen in certain chemical processes. The economists' way of talking account of this has been to visualise substitution, not among inputs in the same process, but to conceptualise a number of 'activities' producing the same product (with fixed input proportions) and then to postulate certain conditions like 'additivity' among processes to indirectly generate possibilities for input substitution. If a sufficiently large number of such activities is presupposed with additivity property, it is presumed, by appropriate combination of activities 'marginal products' could be identified (at least in an approximate sense within limits). These assumptions however are the economists' device to simulate conditions of continuous substitution through convenient assumptions. Such assumptions may not at all rest on technological feasibilities

The Valuation Problem

Moreover, the identification of marginal product in the price-free physical sense is not possible when either the 'fixed' factor or the 'variable factor' or output happen to be value aggregates. The well known question that D.H. Robertson raised regarding the meaning attributable to 'holding the fixed factor constant' when the so-called fixed factor happens to be capital, was to pose significant logical objections to the marginal productivity theory of distribution. Robertson illustrated his case with the example of men digging with spades. In order to find the marginal product of an additional digger (whose 'labour' is qualitatively the same kind as the others) he would have to be equipped with a similar spade, violating thereby the condition of 'amount of capital i.e. the number of spades, held constant. On the other hand if capital is to be treated in terms of value, the same amount of capital (possibly now transformed into inferior spades) must be spread over all the diggers, including the additional one. This raised the question: how does one ascertain the equality of amounts of heterogeneously constituted capital? Being itself a produced commodity, the value of a capital good would vary with a change in the wage (or profit) rate and if the purpose of constructing the marginal productivity schedule of capital was to explain distribution (wages and profits) in the general equilibrium framework then it was clear that the impossibility of measuring capital independently of prices would create a logical difficulty for such an explanation of distribution.

It is not our purpose here to enter into controversies concerning the theory of distribution. Suffice it to point out that the various efforts by economic theorists to construct a consistent measure of capital independently of distribution to generate the requisite 'normally shaped' demand curve for factors of production, particularly capital (based on the diminishing marginal productivity argument) brought to the fore a number of internal contradictions that have successfully challenged that theory.[3]

What is of immediate relevance to our discussion concerning the statistically fitted and the economic production function is the considerable literature that grew up as early as in the thirties around the question of validity and meaningfulness of testing the hypotheses of prevalence of competitive conditions together with marginal productivity theory of wages on the basis of fitted production functions. The provocation for these controversies was provided by Douglas' Theory of Wages and his popularisation of the Cobb-Douglas form of production function. There was an onward rush to use the statistically fitted function, using both cross-section and time series data to test empirically the marginal productivity theory of distribution.[4] Many critics forcefully pointed out the pitfalls in interpreting the statistically fitted function as the function in economic theory and questioned the meaningfulness and the validity of tests. The fundamental objections raised there have been conveniently bypassed. This is but one example of the economic theorists' habit of ignoring basic criticism under the name of pragmatism[5] or even more, by cleverly defending the theory from its logical flaws by assuming away the possibilities of such logical errors through convenient axiomatisation, however unrealistic and therefore devoid of meaning.

In the studies on Indian Agriculture, the marginal productivity schedule has not been used to explain distribution (i.e. wages or profits) as much as to discuss factor utilisation (particularly use of labour) and to discuss efficiency of resource use. Usually in such exercises, factor and product prices are taken as given and not attempted to be themselves expalined. Often, perfect competition is assumed so that these prices are uniformly given for all producers; in some studies, individual cultivators (or group of cultivators) face different prices. The former is the more usual assumption. It is believed that by comparing the marginal value product of labour to wages, for example, one can judge whether or not the particular producing unit is optimally utilising its labour.

Sometimes it is attempted to find out whether, if the equality is not maintained on some farms, the violation is due to systematics causes and whether these causes are associated with the characteristics of the producing unit like its belonging to the small size category or tenant category etc. Even in such uses certain difficulties, apart from the one of valuation referred to above, arise in identifying the marginal product of a factor. We concentrate on labour, as that has been most discussed in the literature.

I.4. 'Labour' as a homogeneous category:

In the empirical studies on Indian agriculture, as elsewhere, marginal productivities of labour and other inputs are not derived from actual experiments of increasing one input, holding the rest at a constant level but from statistically fitted production functions. The usual method adopted is to specify a production function, involving labour, bullock labour, cost of equipment in money terms, irrigation cost, etc., as inputs, each input taken as homogeneous and accurately defined. Errors are expected to be associated only with outputs and the function fitted to cross section data. For the following discussion, we take output as homogeneous so as to concentrate on the problems concerning labour.

Since in agriculture, labour is spread over a long duration and constitutes a number of separate activities following sequentially, a number of problems arise in treating labour aggregatively as a homogeneous input.[6] There are various types of activities like weeding, hoeing, ploughing, sowing, interculture, harvesting etc. each requiring different kind and intensity of labour. Some of these operations require specialised skills and different kinds of associated equipments to carry these out. Also these activities follow in a sequential time-bound fashion.

There would be no particular problem of aggregating these separate labour activities if they were perfect substitutes (which they obviously are not) or they were perfectly complementary. In the latter case, instead of simply adding up labour into one homogeneous magnitude, labour in different operations could be treated as distinct elements and the unit of labour could be defined as a composite unit, as a vector of different labour-uses in different operations. Perfect complementarity i.e. fixed proportionality in the distribution of labour over various activities, would not pose a problem; every 'additional unit of labour' being now treated as an equi-proportional increase in labour input devoted to all the separate activities. However such a heuristically simple solution is not adoptable for the relations among various activities is neither one of substitutability nor one of perfect complementarity. There is scope to vary the relative allocation of labour time as among the activities. It is possible to plough the land a multiple times, deep or shallow; to weed land carefully or otherwise; to irrigate a plot a multiple number of times etc.

It may be suggested that each one of the separate activities could be treated as a distinct use of labour (labour here is analogous to any other commodity with multiple uses). Since in each operation there would be a physical limit beyond which it may not be worthwhile to put in more labour (say ploughing), the producer may be supposed to allocate the labour as among different uses so as to have equal marginal value productivity in all uses. This, then reduces to a standard allocational problem.

Some problems crop up even for such a treatment. In agriculture it is difficult to distinctly associate contribution to yield by particular operations. Since the output emerges only at the end of the production cycle and for many operations there may be no independent measure or value of output, there is a tricky problem of imputing productivity to separate operations. How does one measure the productivity of sowing or weeding? There could be an area-measure (e.g. so much area weeded or sown) but the question then remains: how to compare inter-operation productivities without valuation of these 'outputs'? And, such a comparison would be needed to 'equalise' marginal productivity in all uses.

Further, labour put in one use, A may affect the productivity of labour put to use B. Better weeding and preparation of soil may add to the productivity of labour in sowing or interculture. Heuristically, considering the variety of uses to which labour can be put to and the interdependence in their effectiveness (or productivity), one may yet conceive of a producer taking into account all possible combinations of labour uses and ranking the alternative combinations according to their productivity; so that, depending upon the total labour constraint he moves on to inferior combinations. Hence, theoretically, one may yet have a downward sloping marginal productivity schedule. However it would be a much more complicated construction than the usual one which ignores the variety of uses of labour and the possible reallocation among them affecting yield.

There are further complications, however. Not only there exist various 'uses' of labour but labourers of different categories may be involved in different operations. Thus some operations require skilled labour. Some operations have been customarily, sometimes due to religious and traditional taboos, performed only by certain castes or are restricted according to the sex of the workers. Several authors have remarked about the rather striking feature of the Indian situation that even the very small holdings hire labour despite the fact that their own family labour remains underutilised.[7] The fact that labour is hired, is sometimes erroneously put out as an example of full utilisation of family labour or to argue that the ruling wage rate must be taken as an adequate measure of the 'opportunity cost' of family labour, i.e., it is implied that at the point where the small operator engages hired labour, the marginal productivity of his family labour ought to be equal to the wage rate.[8] Such conclusions implicitly treat all labour as homogeneous and imply that labour could be substituted across 'family' and 'hired' categories. However hiring in of labour may arise due to a number of situations which are compatible with nonfull utilization of family labour; nor does such hiringin validate the 'equalizing of marginal productivities of hired and family labour. Part of the reason why indeed small farmers may hire labour may be the heterogeneity of labour arising out of the specificity of certain operations restricted to only certain categories of workers, who, therefore, need to be hired. Other reasons for hiring labour may also be thought of; such as the paucity of certain equipment (like bullocks or transport) which may necessitate hiring of the complementary labour together with the equipment. In better-wage periods, the small cultivator may hire himself out for a higher wage of longer periods while hiring in labour at a lower wage or briefer periods.[9] The crux of the matter is that labour involved in different operations may not be substitutable and hire of labour may be influenced by a number of other (i.e. apart from wages) economic and noneconomic factors.

I.5. Time - spread of Labour Use:

Another factor that complicates aggregation of labour is its spread over time. Even if labour were not operation-wise heterogeneous the time dimension would create problems on its own. The cultivating household operates under varying constraints in different periods which affects its deployment of labour. Again, if there were perfect complementarity among labour uses over time, we could operate with a vector giving time-distribution of labour, a 'unit of labour' itself being defined in such vectorial terms. Under such a circumstance, the constraints determining the labour use in the most crucial and stringent period (viz. harvesting period) would set the scale for fixing the level of labour use in all other periods.[10] However no such strict complementarity prevails and within limits there is flexibility in varying relative labour use over different periods. On the other hand, the labour use in different periods is neither perfectly substitutable nor totally independent. If it were perfectly substitutable aggregation of labour is possible while if it were independent each period's labour use could have been used as a different labour input with agricultural activities appropriately periodised. We have elsewhere[11] discussed in greater detail how the employment on farm and off farm (i.e. working on one's own farm and hiring out one's labour) are not entirely independent choices and how the level and nature of farm activity interacts with the time-specific 'availability' of employment outside. Similarly the status of the cultivating household significantly determines how far it can exercise free decisions in labour use. A tenant, particularly the petty one, is often committed to rendering unpaid or underpaid labour services to his landlord during specific periods when he is not free to work on his own farm.[12]

Thus by aggregating labour over periods, the time-specificity of such constraints cannot be taken into account.

In short, the rather simplistic view of treating labour as homogeneous and aggregating over operations and time-periods to derive a single marginal productivity schedule ignores some important factors that affect labour-use and the schedule loses its analytical meaningfulness.

I.6. Holding other inputs constant

As already mentioned, it is not very clear as to what one means by holding other inputs constant. A problem concerning valuation of the fixed factor, when it is of heterogeneous composition, has already been referred to. Even if the valuation problem (important for the critique of distribution theory) were set aside, it is still evident that parallel to the variety of uses of labour in different operations is the variety of accompanying instruments and cooperant inputs. A different type of distribution of labour would imply different types and quantities of co-operating inputs In fact in many of the studies, estimating marginal products based on the statistical production function, it has been found that there is a close association between labour and some oth r inputs, e.g., bullock labour.[13] To the extent that there is a high degree of such an association among labour and other inputs, the marginal productivity of any one single input can not strictly be isolated. If the production function, in its specification, drops one of the inputs so related to labour, the estimated marginal productivity of labour would overvalue labour's contribution.[14]

Testing efficiency of Resource Allocation on the Basis of a Statistically Fitted Function:

So far, we have briefly indicated the difficulties involved in treating labour as a homogeneous input and in separating out the contribution to product rendered by exclusive variations in the labour input. These are problems confronting the construction of marginal productivity schedule. However, even if such a schedule could not be directly constructed, it may be indirectly estimated from statistically fitted functions. Certain fresh problems arise when it is attempted to derive theoretical propositions, from the statistically fitted production function treating it as if it were the production function of economic theory.

An exercise frequently undertaken in recent studies on Indian agriculture involves estimating marginal value products from fitted production functions and comparing them with factor prices to test whether the inputs are utilised efficiently or not.[15] Efficiency of input use is adjudged by testing for equality of marginal value product of the same input in different uses and for equality among ratios of marginal value products of individual inputs to their respective prices.[16] In most exercises not only the cross-section data is utilised but it is also presumed that competitive conditions prevail and all the producing units are facing the same technology and prices. This involves a contradiction in itself. For, the cross section data gives the techniques which are in practice, i.e. already chosen by the various producing units on the basis of economic data confronting them. By statistically fitting a production function the different observed techniques are treated as though they were depicting feasible techniques (*ex ante* possibilities) on a single production function, their deviations from the fitted function being assumed as due to errors. Now the competitive assumption of all the producing units maximizing profits and confronting uniform technology and prices should, in theory, yield only one technique as optimum. Therefore in order to judge whether there is in actual practice, efficiency in resource use, the concept of an 'average' farm has to be introduced, and tests to be carried out whether the resource use on such a farm satisfies the efficiency conditions mentioned above. In using the statistically fitted function in the place of the theoretical one, drastic modifications enter at two points: in presuming that the observed techniques are all on the same production function[17] and secondly, that an 'average' farm 'representing' all the actual observations can be meaningfully defined. In the first, points on the statistically fitted function are taken as distinct, theoretically estimated techniques while the actual observations contain an error component (whose distribution is postulated). In suggesting that the observed techniques can be meaningfully represented by a single average technique, it is presumed, on the other hand, that the observed techniques deviate from the average, not systematically but again due to errors. Two, rather incongruous, assumptions are thus involved. First that the observed points can be, within reasonable limits of error, treated as distinct points yielding the estimated production function - the points must be 'distinct' enough to yield the function. Secondly, that they can be, again within reasonable limits of error, also treated as represented by a single point, their deviations from the single point considered as due to statistically permissible errors. In a number of studies,[18] a Cobb-Douglas function is fitted and the marginal value products estimated at the geometric mean. That such a procedure is statistically questionable has been pointed out by A. Rudra in his forceful critique of the allocative efficiency exercises where he raises a number of other related issues as well.[19]

I.8 Theoretical Interpretations on the basis of the fitted function:

A number of theoretical i

of the fitted production funct:

value products of factors to their prices. Rudra, in the paper referred to above, points out how the notion of the 'average' farm used to estimate marginal value products could lead to highly misleading results: For example, every cultivating household may individually fail to satisfy the condition of allocative efficiency and yet, the averaging procedure (the marginal value products estimated at the geometric mean) may not refute such an allocative efficiency hypothesis. A similar criticism may be raised against the interpretation of results obtained over successive years. G.K.Saini[20] finds a situation in which the ratio of marginal value product of labour estimated at the geometric mean to wage significantly exceeded unity in one year and a fall in the ratio followed in the successive year. This he interprets as a move towards greater rationality of resource allocation by the producers. It may be pointed out that given the initial distribution of these ratios over the individual cultivating households, even if each (or a significant majority of them) had moved in a direction contrary to what allocative efficiency principle required in their individual cases, one could yet obtain, on an average, an apparent move in the right direction.

I.9 Doubts about the approach

Apart from the conceptual and statistical difficulties, it is doubtful whether this approach to study rationality of resource allocation can be at all meaningful in the Indian context. In stating the equality of marginal value product of a factor and its price as a condition for efficiency, it is assumed that the price reflects the 'scarcity value' of the resource to the economy as well as to the individual cultivating household. Now, if, in fact, different cultivating households are observed to adopt different methods of production and the 'average' farm appears to be a myth, it is evidently not an accidental phenomenon to be attributed to 'errors' or deviations but due to some systematically operating factors. The different cultivating households with their diverse resource position and their varying economic status arising therefrom face very different kind and range of choices on the market. Moreover the markets in our agrarian economy can be, by no means, characterized as affording equal and uniform opportunity to all. Neither are 'prices' the only basis of transactions nor the participants equally free to make choices. Elsewhere[21] we have noted how the resource position of sections of peasantry affects their market involvement and how, in turn, the peculiar characteristics of market forms and market involvements might constrain production and investment choices. Under such conditions factor and product prices play a varying role for different sections of peasants and are only one element in the process of decision-making.

II

Returns to Scale as Homogeneity condition on the Production Function:

Returns to scale, we noted earlier, has been used also to characterise the scale of the change in output due to proportional expansion of inputs. In fact it is in this usage that the term is now more commonly used. That is, if the theoretical function is $f(x_1, x_2, \ldots x_n) = Y$ we note the value of m in $f(\lambda x_1, \lambda x_2, \ldots \lambda x_n) = \lambda^m Y$. If $m = 1$, i.e. output changes exactly equal to the scalar change in the vector of inputs (or, the function is linear homogeneous) then constant returns to scale obtain. If $m > 1$ (or $m < 1$) then increasing (decreasing) returns to scale obtain. It must be noted that the proposition relates to the expansion path of output across isoquants while the shape of the isoquant is determined by the function $Y = f(x_1, x_2, \ldots x_n)$ itself.

A common theoretical error is to treat 'constant costs' and 'constant returns' in the above sense as identical[22] and as mutually replaceable terms. 'Constant returns' imply constant costs provided all input and product prices remain invariant when there is a change in the scale of operation. This is theoretically the case with regard to the individual producer in a competitive market to whom all prices are given independently of his own production plans. On the other hand, constant costs (in the sense of constant average costs of production with changing output) may prevail even if the production function is not linear homogeneous.

Ideally speaking the hypothesis of returns to scale (constant, increasing or decreasing), should be tested out under conditions of a producing unit actually expanding all its inputs proportionately and the effects on output noted; both inputs and output being individually homogeneous and accurately measured. Such variations to be ideally observed would call for controlled experiments. However the engineering processes may just rule out such possibilities of proportional expansion either as being physically implausible or because the nature of the product itself alters with such changes.

Given the probability of such experimentation the usual procedure has been to use cross-sectional data. Here too, it is highly improbable that different producing units would in fact be observed to use the same input combinations, albeit operating at different scales. Therefore the procedure adopted is to fit a statistical production function, usually the one chosen being of the Cobb-Douglas variety, $Y = A x_1^{\alpha_1} x_2^{\alpha_2} \cdots x_n^{\alpha_n}$ and to test the fitted production function for linear homogeneity. In the case of the Cobb-Douglas function, the hypothesis to be tested is whether the sum of the exponential parameter, $\sum_1^n \alpha_i$, as estimated differs significantly from unity. Testing such a hypothesis in relation to cases drawn from Indian agriculture, some authors have arrived at the conclusion that one may not be far wrong in characterising the production function as being linear homogeneous.[23]

There is however one intriguing result which the authors derive, having asserted that the fitted production function is Cobb-Douglas of the linear homogeneous form. Independently by fitting a regressional relation <u>on the same cross-section data</u> between yield per acre (i.e. productivity of land) and size of holding they obtain statistically what they claim as an evidence for an inverse relation between the two.[24] They argue that this finding is consistent with the fact that the fitted production function $y = A x_1^{\alpha_1} x_2^{\alpha_2} \ldots x_n^{\alpha_n}$ is linear homogeneous (i.e. not significantly different from unity). (For, it is clear that $\frac{\partial y}{\partial x_i} = \alpha_i \frac{y}{x_i}$ and since $0 < \alpha_i < 1$, the marginal product of any input is less than its average product, a case therefore pointing to diminishing returns to a factor as a logical result). In thus clubbing together their statistical result regarding the inverse relation based on the regression exercise with the logical result drawn from the linear homogeneity property there appears to be an inconsistency, which we note in the following. It illustrates the more general point we have already made regarding the fitted function.

While the production function is estimated from cross-section data, competitive conditions are also assumed viz. all producing units face a common technology, the same input and product prices, uniformly free access to markets. If indeed the 'observed' technology is linear homogeneous, there would be a single input combination which is optimum and if inputs are variable for all producing units, as would be the case under perfect competition, the size of operation (or scale of output) of any producing unit is indeterminate. If we take the initial size of holding as the scale-fixing factor then all the producing units would have, allowing for statistically permissible errors, the same input proportions, the level of which would be appropriately adjusted to the size of the holding. In other words, allowing for statistical errors, the input proportions on different producing units must not deviate systematically or significantly and the average productivity of inputs would be expected to be (approximately) uniform on all farms.

Therefore to relate average productivity of any particular input to the size of holding over the same cross-section data and read into the estimated function a systematic inverse relation appears to be inconsistent. If the average input productivities deviate among farms they would be, according to the linear homogeneity hypothesis and perfect competition, due to errors which

cannot then be systematically related to the size of holding, (which here is the scale-determining factor). But the 'inverse relation' precisely does that. A result based upon the theoretical production function seems to have been here carried over to the statistically fitted one. By arguing that the observed data generated under competitive conditions support constant returns to scale, it is being suggested that the observed points lie along the same expansion path while the theoretical proposition concerning the inverse relation relates to movements along isoquants. The same observations cannot be, at the same time, treated as if they were lying on the expansion path and cutting across it.

Further in some studies it is observed that while the Cobb-Douglas production function fitted to the total crop production has exponents whose sum is not significantly different from unity, a similar function when fitted to individual crops gives the sum of estimated exponents significantly less than unity (hence interpreted as indicating prevalence of diminishing returns to scale) and in some other cases greater than unity (hence the technology ascribed increasing returns to scale[25]). If therefore there are cropwise differences then, logically speaking, the 'technology' cannot be discribed as one of constant returns to scale. Rather, one has to explore the reasons to explain cropping patterns which throw up such a result in the aggregate. We shall have occasion to return to this in the following section.

III

'Returns to scale' in some recent discussions of Indian agriculture has been used in a rather loose sense: to denote the relationship between yield per acre (i.e. productivity of land) and the size of holding. In so doing, first, 'returns' are identified with gross productivity of land, which is only one of the several inputs. Secondly 'scale' is represented by the size of holding. Both are particular usages. Scale, it has been suggested, may alternatively be represented by the value of gross output of the size of gross investment on the farm. To the extent that the economic strength of the cultivating household is better measured by these indices, they may be more suitable indicators of scale.[26] Of course, the importance of choosing the value of gross output rather than the size of holding to measure scale would arise when these two indicators are not closely correlated. If they are, either could be used.

A proposition that attracted considerable notice, following the publication of farm management data, is the so-called inverse relation between yield per acre and size of holding. A number of statistical studies carried out regressional fits on cross-section data on holdings and while the statistical results were not entirely conclusive, a number of them did not refute such a hypothesis.[27]

This finding was variously interpreted: some concluded that the 'relative superiority' of smaller holdings could be adduced as a direct support to land reform measures which were expected to lead to distribution of land into smaller holdings.[28] Some interpreting the result as showing absence of economies of scale talked of the greater productive potentiality of 'peasant' farms.[29] An adequate explanation of the 'inverse relation' (if indeed it is a stable and reliable one) is important not only for policy formulation but also for assessing the future possibilities. Much depends upon whether the productivity differentials are a characteristics that could persist, with the small farms proving to be more efficient once or whether their relative advantage rests on a specific conjuncture subject to change, or, whether it is more of a static reflection of the economic vulnerability rather than productive efficiency of the small cultivators

Elsewhere[30] we have discussed in greater detail the alternative hypotheses that have been advanced, mainly in terms of the qualitative superiority of land and labour, higher intensity of cultivation involving greater labour use per acre and the greater proportion of high value crops on smaller farms. We have noted there that neither the qualitative differences in land nor in labour provide an adequate explanation for the inverse relation. It is significant to note that when the yield per acre is related to the size of holding in the case of individual crops no systematic or consistent relation obtains.[31] Also when the yield per acre in the aggregate is related to the gross area cultivated rather than the size of holding (which measures 'net' area), the 'inverse relation' weakens considerably.[32] Both indicate that the differences in value productivity per net acre over farm sizes may be arising due to differences in intensity of cropping and cropping patterns.

The former, namely, the higher intensity of cropping involving application of larger amounts of labour to land on smaller holdings has received prominent attention in the literature. The greater degree of labour use on smaller farms has been explained variously. One of the earliest explanation of Sen[33] was couched in terms of a dual system of farming; the smaller peasant farms being run on family basis, maximizing output and hence using labour upto the point where its marginal productivity is almost zero while the larger holdings, organized capitalistically, and maximizing profits, equating the marginal product of hired labour to wage and hence applying much less labour to land. A more general formulation has been suggested among others by Sen,[34] in terms of the notion of marginal supply price of labour being different in the case of family and hired labour. A number of factors, the most prominent of which are the uncertainty and dearth of employment opportunity and preference of workers for staying on their own farm, according to this explanation, lead to the marginal supply price of family labour being generally lower than that of hired labour.

It is important however to know as to whether the overextended application of labour on the small farms arises due to the positive factor of preference or simply because of lack of employment opportunities. While the former is a factor in favour of self-cultivation, supporting the image of the 'proud and self-reliant' peasant, the latter reflects the compulsion of economic necessity. It appears that the latter is a more significant factor. Moreover, the overextended use of labour on small farms, it is evident from a number of studies, arises due to the pressure of raising as much of subsistence on the tiny plot of land accessible to the culti- vating household. In studies of tenancy,[35] for example, we have noted that the bigger landlords leased out their land in smaller units precisely in order to increase exploitation of the tenant family which is thereby compelled to produce the maximum possible on the tiny leased in plot.

We have elsewhere[36] suggested that the greater proportion of high value-yielding crops produced on petty holdings, leading to the high value productivity of land, may also reflect the peculiar economic status and market involvement of the petty peasantry. The economic compulsion of raising as large a gross income as possible on the tiny holding may induce the small peasantry to over-extend its labour use and also adopt the high gross value-yielding cropping pattern. Incidentally, these crops are also labour intensive, requiring little of additional capital investment but more of circulating capital.

If, therefore, the high intensity of cultivation of land is not so much due to superior techniques and/or better quality of land or labour input and/or the positive preference for or interest in self-cultivation but due to economic stringency, the growth implications of the situation are drastically different. The small farms may not be necessarily 'efficient' in terms of their capacity to generate investment potential. Nor can their 'superiority' be considered as necessarily a long term phenomenon. To ascribe therefore to the 'inverse relation' the character of 'returns to scale' and conclude that there are technological diseconomics of scale would be erroneous. What is noticed is not so much a technological phenomenon as an outcome of certain property relations. In fact, in later studies on farm productivity more attention is being paid to the institutional characteristics of farms, and factors such as whether they are tenanted or self-cultivated holdings, or whether dependent mainly on family or hired labour, have been brought prominently into discussion, moving away from the technological bias of the earlier discussions.

To conclude: In this, rather wide-cast note we have looked at some of the conceptual and theoretical problems that arise in the analysis of the so-called technical relations in agriculture and in the economic interpretations based thereupon. We have indicated the frequent source of confusion in interpreting statistically fitted functions as if they were the function in economic theory. We have, using specifically the notions of 'returns to a factor' and 'returns to scale' much in vogue in agricultural economics illustrated these conceptual and statistical problems and cast doubt on the economic interpretations given to the empirical findings.

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Footnotes

1. See for a different method for estimating the production surface: Farrell M.J. "The Measurement of Productive Efficiency", Journal of Royal Statistical Society, 1957 and Farrell M.J. and Fieldhouse M: Estimating efficient production function under increasing returns to scale". The Journal of the Royal Statistical Society, Series A, 1962., wherein only the technique with the minimal input combinations define the surface. While this method of estimating the production function is closer to the theoretical definition, it has certain drawbacks, one of which is its extreme sensitivity to inclusion or exclusion of the lower extremal points. A.K. Hati and Ashok Rudra in their "Calculation of Efficiency Indices of Farmers" (Economic and Political Weekly, March 31, 1973) have suggested a variation on the usual regression method to accommodate the economic theoretic property of the production function, while avoiding the extreme sensitivity of the Farrell method.

2. This was the basis of the so-called 'Ricardian theory of rent'. For pointing out the specific nature of its context in relation to distribution theory, see P. Sraffa: "The Laws of Return under competitive conditions", Economic Journal, 1926. See also K. Bharadwaj "Classical Political Economy and the Rise to Dominance of supply and Demand Theories" (unpublished), R.C. Dutt Memorial Lectures, Calcutta, April 1976.

The Debates in capital theory have been covered by Harcourt G: Some Cambridge Controversies in Capital Theory (Cambridge University Press, 1969). See also "Paradoxes in Capital Theory: A symposium", Quarterly Journal of Economics, 1966.

A spurt of literature on testing marginal productivity theory of distribution using the Cobb-Douglas function followed the publication of P.H. Douglas' Theory of Wages, New York, 1934. See Bronfenbrenner "Production Function; Cobb-Douglas, Interfirm and Intrafirm", Econometrica 1944; Handsaker and Douglas: "The Theory of Marginal Productivity tested by data for manufacturing in Victoria", Quarterly Journal of Economics, 1937-38, Bronfenbrenner and Douglas, "Cross-Section studies in the Cobb-Douglas Functions", Journal of Political Economy, 1939; Gunn and Douglas "The Production Function for American Manufacturing, 1911", American Economic Review, 1941: An Excellent critique was presented by H. Menderhausen in "On the significance of Professor Douglas' Production Function", Econometrica, 1938. Also critical articles appeared by David Durand: "Some Thoughts on Marginal productivity with special reference to Professor Douglas' analysis", Journal of Political Economy, 1937; Victor Smith "The Statistical Production Function", Quarterly Journal of Economics, 1944-45 and discussion lately revived by Phelps-Brown E., "The Meaning of the Fitted Cobb-Douglas Function", Quarterly Journal of Economics, 1957.

5. Cf. "I have never thought of the macroeconomic production function as a rigorously justifiable concept. In my mind, it is either an illuminating parable or else a mere devise for handling data, to be used as long as it gives good empirical results and to be abandoned as soon as it does not or as soon as something better comes along". R. Solow in his review of Hicks' Capital and Growth in American Economic Review, 1966.

6. See, among others, Bhagawati and Chakravarti "Contributions to Indian Economic Analysis", American Economic Review, 1969; Krishna Bharadwaj: "Production Conditions in Indian Agriculture", (Cambridge University Press, 1974) and A. Rudra and M. Mukhopadhy: Hiring of Labour by poor peasants" Economic and Political Weekly, Jan. 10, 1976.

7. Cf. footnote 6 above.

8. See Bhagawati and Chakravarti: "Contributions to Indian Economic Analysis", American Economic Review, 1969, Supplement Vol.59 and Krishna Bharadwaj: "Production Conditions ....etc". op.cit. p.27.

9. See Krishna Bharadwaj op.cit. p.25-26.

10. Sen A.K. Employment Technology and Development (Oxford University Press. 1975); Stiglitz J.E. "Rural-Urban Migration, surplus labour and the relationship between urban and rural wages" quoted in A.K.Sen op.cit.p.74.

11. Krishna Bharadwaj op.cit. p.21-23.

12. Krishna Bharadwaj and P.K. Das: "Tenurial conditions and the mode of exploitation in some villages of Orissa". Economic and Political Weekly. Feb. 75.

13. P.K. Bardhan: "Size, productivity and returns to scale; an analysis of farm level data in Indian agriculture", Journal of Political Economy, Vol.81, 1973.

14. A.K. Sen: Employment, Technology and Development, op.cit. p.151.

15. Among others, Chennareddy V: "Production efficiency in South Indian Agriculture", Journal of Farm Economics; Nov. 1962; Dillon and Anderson "Allocative Efficiency, traditional agriculture and risk", American Journal of Agricultural Economics, Feb. 1971: Hopper D. "Allocative Efficiency in a traditional Indian agriculture" Journal of Farm Economics, 1965; Sahota G.S. "Efficiency of resource allocation in Indian agriculture". American Journal of Agricultural Economics, 1968; Saini G.R. "Resource use efficiency in Agriculture", Indian Journal of Agricultural Economics, 1968; Srivastava and Nagardeva "On the allocative efficiency under risk in transforming traditional agriculture", Economic and Political Weekly, 1972 and Wise and Yotopoulos: "A test of hypothesis of economic rationality in a less developed economy", American Journal of Agricultural Economics, 1968.

16. Let us suppose, there are N farms, n products and m inputs. Usually the Cobb-Douglas model used to estimate the production function. $Y_j = A_j x_{ij}^{b_{ij}} e_j$ where e_j is the error term, Y_j is the j^{th} output and x_{ij} the i^{th} input in producing j^{th} output. If m_{ij} is the marginal product evaluated at the geometric mean of observed x_{ij}s over the N farms, (i.e. $m_{ij} = \frac{\partial Y}{\partial x_{ij}}$ at the geometric mean of x_{ij}) then a test is carried out to see whether $p_j m_{ij} = p_s m_{is}$ or $p_j m_{ij} = p_i$ where 'p' denotes prices with the subscript denoting the relevant commodity

17. Victor E. Smith: "The Statistical Production Function" op.cit.

18. See footnote 15 above.

19. A. Rudra: "Allocative efficiency of Indian Farmers: Some Methodological Doubts", Economic and Political Weekly, Jan. 70.

20. G R. Saini: "Resource use efficiency in Indian Agriculture" op.cit.

21. K. Bharadwaj: Production condition in Indian agriculture op.cit.

22. A.M. Khusro: "Returns to Scale in Indian Agriculture", Indian Journal of Agricultural Economics, 1964.

23. See Khusro A.M. op.cit.; Saini G.R.: "Holding size, productivity and some related aspects of Indian agriculture", Economic and Political Weekly, June 26, 1971.

24. For example, See G.R. Saini: "Holding size....etc" op.cit.

25. G.R. Saini: Economics of Farm Management with special reference to some reference to some selected holdings in U.P." (mimeographed).

26. U. Patnaik: "Economies of Farm size and Farm Scale", Economic and Political Weekly, Special No.1972.

27. See for an exhaustive bibliography A.K.Sen: Employment, Technology and Development op.cit. Also K. Bharadwaj: "Notes on Farm size and productivity", Economic and Political Weekly, March 1974.

28. For example, C.H.H.Rao: "Alternative Explanations of the Inverse Relationship between Farm size and output per acre in India", Indian Economic Review, 1966. Also Keith Griffin: Political Economy of the Agrarian Change (MIT Press, 1974)

29. Following the arguments of Chayanov A.V. "The Theory of Peasant Economy (ed) Thorner, Kerblay and Smith, 1966 and Georgescu-Roegen N. "Economic Theory and Agrarian reforms", Oxford Economic Papers, 1960.

30. K. Bharadwaj: "Production Conditions in Indian Agriculture, op.cit.

31. A. Rudra, "More on returns to scale in Indian agriculture", Economic and Political Weekly, 1968. and K. Bharadwaj: "Notes on size and productivity" op.cit.

32. C.H.H. Rao: "Farm size and Economies of Scale", Economic Weekly, 1963.

33. A.K.Sen. "An aspect of Indian Agriculture" Economic Weekly, 1962.

34. A.K.Sen: "Peasants and Dualism with or without surplus labour", Journal of Political Economy, 1966. Mazumdar D: "Size of Farm and Productivity: A problem of Indian Peasant agriculture", Economica, 1965 and A.K.Sen: Employment, Technology and Development.

35. K. Bharadwaj and P.K. Das: "Tenurial Conditions and the Mode of Exploitation in some villages of Orissa", Economic and Political Weekly, Feb. 1975.

36. K. Bharadwaj: "Production Conditions in Indian Agriculture, op.cit.