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The present study is a revised version of the paper presented in an Open Seminar at CDS on 25 March 2011. I am grateful to the participants and the Chairperson Sri VP Joy IAS, Chairman, Kerala State Electricity Board for encouraging comments and also to Rju for smiling away my excuses for my absences from her little kingdom. Also acknowledged is an anonymous referee at CDS.

ABSTRACT

Time-of-day (peak-load) pricing of electricity is an indirect form of load management that prices electricity according to differences in the cost of supply by time of day and season of year. It reflects the costs in a more accurate manner than do the traditional block rate structures, as it logically stems from the marginal cost pricing theory, yet is compatible with the historical accounting costs. It has long been argued and advocated that the sale of electricity and other services, in which periodic variations in demand are jointly met by a common plant of fixed capacity, should be at time-differential tariffs. Despite a very rich tradition of modeling, theoretical refinements in peak load pricing have not attracted much attention of late. The present study seeks to model seasonal time-of-day pricing of electricity for two types of power systems – pure hydro and hydro-thermal under four structural welfare assumptions – first-best, second-best, monopoly and constrained monopoly, in conditions of both determinism and uncertainty.

Keywords: Time-differential pricing, first best, second best, monopoly, uncertainty

JEL Classification: C6, D4, L94, Q4.

 "*I shall make electricity so cheap that only the rich can afford to burn candles*."

- Thomas Alva Edison

1. Introduction

Three distinct functions are involved in supplying electricity in its usable form to the customers: generation, transmission and distribution, corresponding to production, transportation to market and retail distribution of many other products, the chief differences being that i) electricity moves from the generator to the end-use equipment in a continuous flow at a speed approaching that of light, and ii) it cannot be stored in its original form.

Generation, the production of electrical energy from mechanical energy, takes place at central stations normally far away from consumers necessitating the other two processes of transmission and distribution. Transmission is the moving of this electrical energy from generating plants through wire at high voltage to bulk delivery points called substations where it is transformed down to low voltage ready for distribution through low voltage lines to individual meters.

There are only two basic sources for driving electric generators: hydro and thermal. The energy source in hydro-plant is water-driven turbines and that in the thermal plant, steam-driven turbines, the steam being produced either by burning fossil fuel (coal, oil or natural gas) or by a nuclear reactor.

Electric utility

The electric utility is unique in that its product is one that must be generated at the instant it is to be used. If the utility has excess generating capacity, it can usually meet any anticipated demand; but an overabundance of excess capacity entails increasing cost for idle hours. At the same time few products have a greater need for quality and reliability — cases of brown-outs and black-outs. As a matter of practical economics, electric power systems are so designed as to keep both the black-outs and brown-outs within tolerable limits by means of reserves.

One of the very important components of the electric power system is the customer's load which varies greatly at random according to time of day, day of week and season. A graph showing the variation in the demand for energy along time is called a load curve. From the load curve is derived load duration curve (LDC) defined as showing the amount of time that any given overall load level equals or exceeds a given capacity level. The LDC is one of the most important tools in electric power system planning and analysis.

Tariffs

Tariff is the rate of payment or schedule of rates on which charges to be recovered from the consumer of electrical energy are computed. A number of tariff structures has been designed and put in use with various types of consumers. Usually cost differences have been the primary justification for rate structure differences. The traditional approach involves division of costs into three categories:

- i) capacity, demand or load costs,
- ii) energy (unit), output or volumetric costs, and
- iii) consumer costs.

The first of these ('kilo watt (kw) costs'), related to investments in generation, transmission and distribution, vary with the speed and time with which customers use electricity. The second ('kilo watt hour (kwh) costs') vary directly with the number of units generated; they are mainly fuel costs and operating and maintenance ($\alpha \& m$) costs. And the last are those costs varying directly with the number of customers served rather than units consumed. They include expenses on connection, meter reading, billing, collection and consumer services. Then prices are set so as to recover historical (accounting) costs over these three categories with the 'fair' contribution from the several customer classes usually grouped in terms of diversity and load factor.

This backward-looking embedded (accounting) costs approach, concerned mainly with recovering sunk costs, ignores some very vital issues especially from the angle of efficient resource allocation. The prices should be related to the true value of additional resources required for an extra unit of supply and this necessitates a forward-looking estimate, i.e., pricing according to marginal costs (MC), which are calculated on the basis of expansion plans and operating schedules of the power system in line with demand variation.

Seasonal Time of Day Pricing

The spectre of rising electricity costs can be held in leash to a certain extent through load management of electricity usage, including direct (mechanical) controls on end-use equipments and time-differential tariffs. Loadmanagement meets the dual objectives i) of reducing growth in peak load, thus nipping the need for capacity expansion, and ii) of shifting a portion of the load from the peak to the base-load plants, thereby securing some savings in peaking fuels. By moving toward achieving these objectives electric utilities stand to win a cut in operating and capacity costs, share the gain with the consumers and provide a partial solution to the country's energy dilemma.

Time-differential (peak-load) pricing of electricity is an indirect form of load management that prices electricity according to differences in the cost of supply by time of day and season of year. It reflects the costs in a more accurate manner than do the traditional block rate structures, as it logically stems from the marginal cost pricing theory, yet is compatible with the historical accounting costs. Again, compared to the block rate structures, the seasonal time-of-day (STD) pricing offers more potential for improving system load factors; its cost-based price signal motivates customers to modify their usage patterns, which in turn will move the system toward attaining the above twin goals.

It has long been argued and advocated that the sale of electricity and other services, in which periodic variations in demand are jointly met by a common plant of fixed capacity, should be at time-differential tariffs. Implementation of peak-load pricing involves substantial capital expenditure in changing meters and increasing customer service as well as transition costs of moving from one rate schedule to another. STD electricity rates have widely been in use in some of the advanced countries for several decades to reflect such peak-load cost variations, initially for large industrial customers where metering costs constitute a trivial fraction of the total electric bills.The reforms in the electricity sector have given a fillip to this initiative as spot markets for electricity have come up, rendering the price of electricity on the wholesale market to vary each hour and thus opening up opportunities for electricity distribution companies to apply a real-time pricing scheme to the customer. The progress in solid state technology has now introduced *smart meters* with many advantages over simple automatic meter reading, such as real-time or near real-time readings, power outage notification, and power quality monitoring. The smart meters have now helped these countries to extend STD pricing to almost all consumers.

What follows is divided into four sections. The next section briefly discusses the salient features of the generally accepted welfare models in the context of pricing: marginal cost or first best pricing and monopoly pricing and their constrained cases, second best or Ramsey pricing and regulated monopoly pricing. The third section illustrates the basic peak load pricing theory and the fourth one goes into the modeling of STD pricing of electricity for two types of power systems – pure hydro and hydro-thermal under four structural welfare assumptions – first-best, second-best, monopoly and constrained monopoly. The model is also solved for conditions of uncertainty in the presence of outage costs, included in the objective function of the model. The last section concludes the study.

2. The Welfare Foundations: A Review

2.1 Marginal Cost Pricing

Historically the use of gross surplus as a measure of welfare¹ was apparently first proposed by Dupuit (1844) while evaluating public works projects. The concept was developed and extended by Marshall (1890) and later fructified in Hotelling's (1932, 1938) proposals on public utility pricing.²

As indicated above, the traditional measure of welfare used in evaluating public utility policies has been

 $W = TR + CS - TC$, ….(2.1)

^{1.} Although there have been detractors (e.g., Samuelson, 1947; Little, 1957; Silberberg, 1978; and Bos, 1986), the use of surplus is widespread in applied welfare economics (e.g., Mishan, 1971 and 1981). Willig (1976) has given further justification for its use by demonstrating, under conditions quite reasonable for the utility sector, that consumer surplus closely approximates the consumer benefit in money terms.

^{2.} Traditional interest in the efficiency issues sprang up from pricing aspects only. Later on the realm of efficiency concerns has broadened to involve such considerations as X- efficiency (Leibenstein, 1966) and transaction costs (Williamson, 1975).

where $W =$ net social benefit, $TR =$ total revenue, $CS =$ consumers' surplus, and $TC = total \, costs.$

Now $TR + CS$ is equal to the area under the uncompensated demand curve. Let $p(x)$ be the inverse demand function and $C(x)$, the total cost function. Then we have,

$$
W = \int_0^x p(y) dy - C(x)
$$
(2.2)

The chosen objective is to maximize *W* subject to any constraint relevant for first-best situation, such as the availability of resources and the community's production function.

Maximization of *W* leads to $p(x) = dC/dx$, i.e., price = marginal cost (MC).

One basic deficiency from which our *W* in (2.2) suffers is the independent demand assumption (Pressman 1970).With this assumption, *W* for each good or service can be calculated separately and their sum gives the total net welfare:

$$
W = \sum_{i=1}^{n} w_i = \sum_{i=1}^{n} \left(\int_0^{x_i} p_i(y_i) dy_i - C(x_i) \right) \qquad \dots (2.3)
$$

When we consider the change in the price of more than one commodity, the definition of gross surplus is somewhat more complicated (Hotelling 1932 and Pressman 1970). Let $\mathbf{x} = (x_1, \ldots, x_n)$ represent a typical commodity bundle. Also let $x(p) = (x_1(p), \ldots, x_n(p))$ be the *n* demand function for *x* and $p(x) = (p_1(x),...,P_n(x))$ be their inverse demand function. In this multi-product case the net social welfare at the vector of outputs $x = (x_1, ..., x_n)$ would be (2.3).

But in general, because of the substitute/complement property of products, p_i may be expected to depend on the entire output vector x , rather than just on x_i , as in (2.3) . For the multi-dimensional welfare

function with dependent demands, Hotelling (1932) has suggested a line integral of the form:

$$
W = \oint_0^x \left[\sum_{i=1}^n p_i(y) \mathrm{d}y_i \right] - C(x) \tag{2.4}
$$

where o is some designated path connecting the origin (of *n*-space) and the output vector.

Two major difficulties now crop up with this formulation. First, differentiability of *W*, and second, *W*, as it is now defined, depends on the particular path ο chosen and is thus not unique (Pressman 1970). Thus an indeterminacy arises with variation of the value of the integral when the path of integration between the same end points is varied. The condition that all these paths of integration shall give the same value, i.e., the condition that *W* in (2.4) will depend only on *x* and not on the path is that the 'integrability conditions', invoked from the Independence of Path Theorem for line integrals,

$$
\frac{\partial p_i}{\partial x_j} = \frac{\partial p_j}{\partial x_i}, \ \forall i, j, \tag{2.5}
$$

are satisfied. Hotelling (1932, 1938) has shown that there is a good reason to expect these integrability conditions to be met, at least to a close approximation, in an extensive class of cases (See also Pressman 1970, and Crew and Kleindorfer 1979).

Thus, with the integrability conditions, the line integrals of the form (2.4) become differentiable and their value, *W*, independent of the path σ ; so that the first-order conditions for maximizing *W* in (2.4) again lead to marginal cost pricing.

2.2 Second-Best Dilemma

Though marginal cost pricing has got strong argument appeal, it is not without significant problems. First, departures from marginal cost pricing in some sectors of the economy owing to the immutable violation of any of the competitive equilibrium conditions in those sectors pose serious questions against thieving Pareto optimality in the other sectors of the economy. Such violations in the first-best atmosphere accumulate as what are termed 'second-best' problems. Some of the early contributors on second-best, Lipseyand Lancaster (1956), for example, argue that "To apply to only a small part of an economy welfare rules which would lead to a Paretian optimum if they were applied everywhere, may move the economy away from, not toward, a second-best optimum position" (Lipsey and Lancaster 1956:17).

Later developments, however, have been more positive. Farrel (1958), for example, argues that the second-best optimum is likely to be close to the first-best optimum, implying that price should be set at least equal to MC, and in the case of substitutes, above MC. It has also been pointed out that first-best rules may be optimal even with the particular Lipsey-Lancaster formulation of the second-best problem (see Santoni and Church 1972; Dusansky and Walsh 1976; and Rapanos 1980). Davis and Whinston (1965) indicate that in the face of separability or little or no interdependence between sectors, first-best conditions are optimal in the competitive sectors even when they turn out to be unattainable in the other sectors (see also Mishan 1962).

Lancaster (1979) has later on summarized the whole second-best arguement in the context of the electric utility industry. The small size of individual regulated industries in relation to the whole economy entails a very large manipulation of these sectors in order to counterbalance the distortions of the economy. Since all the regulated industries could not be under a common control, the alternative appears to be to optimize in individual sectors.3"Unless a simultaneous second-best

^{3.} This, in effect, seems to take us back to the case-by-case approach of applied welfare economics used by Meade and others in the beginning of the 1950s and represented in later and technically more elaborate studies by, e.g., Boiteux (1956); Rees (1968); and Guesnerie (1975).

solution is determined for the complete regulated sector, therefore, it would seem that the next best thing (the 'third best'?) isto ignore secondbest elements in pricing policy at the decentralized level."(Lancaster 1979:93).

But still another critical problem remains there – the problem of decreasing costs even if costless regulation could enforce marginal cost pricing policy. The traditional approach, as explained above, defines a natural monopoly in terms of everywhere decreasing average cost curve. Let $AC(x)$ denote average costs, $C(x)/x$, and $MC(x)$, marginal costs, $dC(x)$ dx. Then it can be shown that $dAC(x)/dx = [MC(x) - AC(x)]/x$, so that for any positive output level *x*, if $dAC(x)/dx < 0$, then $MC(x) < AC(x)$. Also if $MC(x)$ is everywhere decreasing (concave costs), then assuming $C(x) \geq 0$, we have $MC(x) < AC(x)$. Thus either decreasing average or decreasing marginal costs lead to marginal costs being less than the average. This results in incurring deficits under marginal cost pricing posing many a problem.4 Attempts to have recourse to taxation for covering deficits will only lead to significant allocative distortions.

Discussions upon the issue of decreasing costs have converged on two alternatives, fair rate of return regulation and welfare optimal break-even pricing.

2.3 Monopoly Pricing

First consider the case of a profit maximizing monopolist who would set price and output such as to

$$
\text{Max}_{x\geq 0} \,\pi(x) = \sum_{i=1}^{n} x_i p_i(x) - C(x) \quad \text{.... (2.6)}
$$

^{4.} The very existence of MC pricing equilibria is challenged (Beato, 1982; and Cornet, 1982). Moreover, the optimality of MC pricing also is challenged (Guesnerie, 1975; Brown a 2nd Heal, 1979, 1980 a and b); Tillmann, 1981). If the production possibilities are non-convex, MC equilibria may fail to be Pareto optima. Though many an attempt has been made to find conditions under which at least one equilibrium is Pareto efficient, there exist examples showing that even in very simple cases such conditions cannot be found (see Brown and Heal, 1979).

This leads to the familiar result that $MR = MC$, i.e., ∂ $R(x)/∂x_i = ∂C(x)/∂x_i$, where $R(x) = ∑x_i p_i(x)$, or from (2.6),

$$
p_i(x) + \sum_{j \in N} x_j \frac{\partial p_j(x)}{\partial x_i} - \frac{\partial c(x)}{\partial x_i} = 0, i \in N = (1, ..., n) (2.7)
$$

$$
\Rightarrow p_i = \frac{MC_i}{1 + \sum_{j \in \mathcal{N}} \frac{R_j}{R_j} \eta_{ji}}, \quad i \in \mathbb{N}; \quad \dots (2.8)
$$

where_I $\eta_{ji} = \frac{\partial p_j x_i}{\partial x_i p_i}$ is the 'flexibility' of p_j w.r.t. x_i (see Rohifs 1979)

and $R_i = p_i x_i$ is the revenue from product *i*. When cross price elasticities of demand are zero, we get the inverse elasticity rule (see Samuelson 1972), pregnant with price discrimination potential.

Depending on the sign of $\partial P_i/\partial x_i$ in (2.8), various possibilities arise; but the usual presumption favours own effects, ∂*Pi* / ∂*xi*< 0, to dominate cross effects, ∂*pj*/∂*xi*, such that the second term there would be negative, resulting in higher prices $p_i(x)$ and lower output x than under MC pricing.

2.4 Regulated Monopoly Pricing

The welfare losses due to monopoly pricing may be limited by regulating5 the level of profits to some 'fair' level, say, high enough to pay at competitive rates the various factors used, including capital. Assuming a fair returns, larger than the market cost of factors *k*, the rate of return regulation may, in general, be captured in the constraint,

$$
\sum x_i P_i(x) - \alpha C(x) \le 0, \tag{2.9}
$$

Where $\alpha = s/k > 1$. Inclusion of this constraint in the above monopoly pricing model yields the optimal prices,

^{5.} Bailey (1973) and Sheshinski (1971) have examined the welfare implications of increased regulation.

$$
p_i = \text{MC}_i \left[1 - \frac{\lambda}{1 - \lambda} (\alpha - 1) \right] / \left[1 + \sum_{j \in N} \frac{R_j}{R_i} \eta_{ji} \right], \quad i \in N; \quad ...(2.10)
$$

where λ is the shadow price of a rupee of profit regulated. In contrast to the unconstrained monopolist who equates MR and MC, the monopolist under rate of return regulation sets MR equal to something less than MC, the deduction being determined by λ and α . The limiting cases refer to zero profits ($\lambda = \alpha = 1$) and to monopoly profits ($\lambda = 0$).⁶

2.5 Ramsey Pricing

The second approach, originated with Ramsey (1927) and developed mainly by Boiteux (1956) and Baumol and Bradford (1970), deals directly with the deficit problem by allowing optimal departures from MC pricing such as to break even. This optimal departure is obtained by maximizing the welfare function (2.4) subject to an explicit breakeven constraint:

$$
\pi(x) \ge \pi_0(x) \tag{2.11}
$$

where $\pi(x)$ is as defined in (2.6) and π_0 is the required profit level.

Assuming the integrability conditions to hold, the optimal, secondbest prices derived are:

$$
p_{i} = \text{MC}_{i}(1+\gamma)/\left[1+\gamma(1+\sum_{j\in N}\frac{R_{j}}{R_{i}}\eta_{ji})\right], \quad i \in N; \tag{2.12}
$$

where γ is the shadow price of a rupee of revenue raised. It may also be written as:

$$
\left[\frac{p_i - \text{MC}_i}{p_i}\right] S_i = -\rho \,, \quad i \in N; \tag{2.13}
$$

^{6.} Though regulation may be able to reduce the abuse of monpoly power, it is fraught with a lot of knots in the context of privately owned public utilities, e.g., Averch-Johnson effect (see Averch and Johnson, 1962) and the tar baby effect. (McKie, 1970). Also see Crew and Kleindorfer (1986, ch.8) for a discussion on the tar baby effect in electricity regulation in private enterprise economies.

where
$$
\rho = \frac{\gamma}{1+\gamma} \ge 0
$$
 is the 'Ramsey number' and $S_i = \frac{1}{\sum_{j \in N} \frac{R_j}{R_i} \eta_{ji}}$

is the 'super elasticity' of x_i (see Rohlfs 1979).⁷ ρ is positive except at the welfare optimum, where $\rho = 0$, and the conditions for the profitmaximizing solution are identical to the above with $\rho = 1$.

Hence a regulated monopoly under Ramsey pricing regime behaves as if it were an unconstrained profit maximizing monopolist faced with a demand curve whose elasticity is inflated by the factor $1/\rho = (1 + \gamma)/\gamma$. It must be noted that if we neglect all cross-price elasticities of demand, the Ramsey price structure reduces to the 'inverse elasticity rule':

$$
(p_i - \text{MC}_i)/p_i = -\rho / e_{ii}, \quad i \in \mathbb{N}; \tag{2.14}
$$

where e_{ii} is the own price elasticity. The price-cost margin of a product is larger, the smaller the absolute value of its price elasticity. The normal own-price elasticity of demand being negative, the Ramsey pricing in general results in positive price cost margins. Under 'low pricing procedures', $\rho < 0$, and we have the case of negative price-cost margins. The positive price-cost margins lead to higher prices of priceinelastic goods and to lower prices of price-elastic goods.

The reverse holds in the case of negative price-cost margins. Thus, in general, the poor who are comparatively price inelastic are burdened in the case of positive price-cost margins and favoured in the negative ones.⁸

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^{7.} It should be noted that if we defined the net social benefit function over the 'budget space', the optimal solution would be in terms of the usual cross-price elasticity of demand, ε _{*ii*}, which can be easily interpreted. Note that $\eta_{ji} \neq l/\varepsilon_{ij}$, and the interpretation of S_i and therefore (2.13) becomes complicated. In fact, \mathcal{E}_{ij} and η_{ij} need not even have the same sign; see Nguyen and MacGregor-Reid (1977).

^{8.} See, for equity aspects of pricing, Feldstein (1972 a, b, c) and Wilson (1977).

3. The Peak-Load Pricing Theory: A Review

Apparently, the first pace of exploration into the basic ideas of peak-load pricing started with Boiteux (1949) of *Electricite de France*. 9 In the USA it was independently originated by Steiner (1957) and developed by Hirshleifer (1958) and Williamson (1966). While Boiteux and Steiner assumed two equal periods, Williamson showed how to work out with periods of any length. Steiner interpreted his peak-load pricing results in terms of price discrimination. Hirshleifer, taking issue with this, suggested that they could be more usefully interpreted in MC pricing terms.

The additional contributions made include Buchanan (1966), Turvey(1968 a, b, c, 1969, 1971), Pressman (1970), Mohring (1970), Littlechild (1970 a, b), Crew and Kleindorfer (1970), Bergendahl (1970, 1974, 1975), and Bailey (1972). The major result common to all these works is that peak-load price should equal marginal peak running costs plus marginal capacity costs, while off-peak price equals only marginal off-peak running costs, since the peak consumers, not the off-peak ones, are solely responsible for raising the 'capacity lid'.

The first major extension to the basic model was provided by Pressman (1970) who synthesized the earlier works by the MC pricing school (for example, Hotelling (1932), Dreze (1964) and Nelson (1964))in constructing a peak-load pricing model with timeinterdependent demands and a more general specification of technology. Crew and Kleindorfer (1971) presented a further theoretical generalization by looking for the implications of a diverse technology (i.e., multiple plant types) for pricing and capacity decisions. Dansby (1975), based on the same technology specifications as Crew and

^{9.} However, according to Ault and Ekelund (1987), the theory of peak load pricing goes back at least to the work of Bye (1926, 1929), who first developed the peak load model.

Kleindorfer (1975 a, b), allowed demand to vary continuously with time within each of the finite number of pricing periods.

Bailey and White (1974) set up a scenario of reversals in peak and off-peak prices as enacted by a monopoly, a welfare maximizing firm with increasing returns to scale, a monopoly under rate of return (RoR) regulation and a firm with a two-part tariff. Their results implied, inter alia, that for customer changes of almost the same size, regulatory authorities with tighter RoR regulations might encourage lower usage prices to peak business users of electricity leaving the prices to off-peak residential users substantially unchanged.

Panzar (1976) presented a reformulation of the peak-load problem in which technology was specified through a neo-classical production function. The best-known result that optimal peak- load pricing requires only those consumers who utilize plant to capacity to bear the marginal capacity costs was shown to result from the fixed proportions technological assumptions of the traditional literature and not from the fundamental nature of the peak-load problem.When a neo-classical technology was specified, it was found that optimal pricing required consumers in *all* periods to contribute towards the capacity cost.

3.1 A Basic Peak-Load Model

Steiner (1957) has adopted the conventional welfare maximizing approach. He assumes a typical 'day' divided into two equal-length periods, each with itsown independent demand curve. Costs are assumed to be linear: *b* is operating cost per unit per period and β the unit capacity cost per day. Neo-classical substitutability between variable and capital costs is ignored. This and the single technology are the critical assumptions that yield 'Steiner's results' for the finite period case.

Now the welfare maximizing problem may be written as $W = \sum_i \int_0^{q_i} p_i(y_i) dy_i - \beta q_n - b \sum_i q_i,$ $i = p, o;$ (3.1)

where q_p and q_o are demands in the peak (q_p) and off-peak (q_o) periods respectively, with peak period demand equalling capacity, and $p_p(q_p)$ and $p_o(q_0)$ are prices in the peak (p_p) and off-peak (p_o) periods respectively.

The corresponding optimal prices are then given by:

$$
p_p = b + \beta
$$
 and $p_o = b$, ... (3.2)

which indicate that peak price covers both the marginal capacity and operatingcosts, whereas off-peak price just covers marginal operating costs. Moreover, it is clear that if there are constant costs, welfare maximization automatically requires the peak price to be higher than the off-peak one.

3.2 Peak-Load Pricing Under Uncertainty

All the above models assume that demand is deterministic. But in general, many public utilities face demands that are not only strongly periodic as in the peak-load model but also stochastic. After the contributions of the French economists discussed by Dreze (1964), Brown and Johnson (1969) sparked off a new controversy as to the effects of stochastic demand on public utility pricing. Brown and Johnson used the familiar cost assumptions of the Boiteux-Steiner-Williamson peakload model, but with a one-period stochastic demand. Their expected welfare maximization yielded the optimal solution as $p = b$, in stark contrast to the corresponding one period deterministic solution of $p = b + \beta$.

Moreover, there lurked at their optimal solution a possibility of excess demand to occur frequently. Turvey (1970) criticized¹⁰ this low level of reliability at optimum as implausible, which spurred Meyer (1975) to reformulate the Brown-Johnson model by adding reliability

^{10.} Salkever (1970) also joined issue with Brown and Johnson in *American Economic Review*

constraints to it; this, in turn, raised a new issue as to determining the optimum levels of such constraints. Carlton (1977) and Crew and Kleindorfer (1978) tried on this issue, still leaving much to be resolved.

Rationing in the event of demand exceeding capacity was another vulnerable point in Brown-Johnson model (Visscher 1973).They assumed a zero-cost rationing process in accordance with the willingness to pay of the consumers, which appeared highly implausible. Crew and Kleindorfer (1976) subsequently examined the simultaneous effects of a diverse technology, Stochastic demand and rationing costs on the peak-load pricing policy of an expected-welfare maximizing public utility. Both uncertain demand and uncertain capacity were considered simultaneously in a simple model by Chao (1983). He examined demand uncertainty in a more general framework within which the hitherto specifications of demand uncertainty, in either additive or multiplicative form, were seen as special cases. The work took explicit account of the random availability of installed capacity, a major source of uncertainty contributing to electricity supply shortages.

The theoretical refinements have not attracted much attention of late, possibly because the classical framework and the inevitable result have been taken for granted, and the research interest has shifted from theory to empirics. However, Pillai (2003) has taken up the basic peak load model to question the classical framework and its result and shown that if the off-peak period output is explicitly expressed in terms of capacity utilization of that period, the result will be an off-peak price including a fraction of the capacity cost in proportion to its significance relative to total utilization. Analyzing the implications of the relationship between reliability and rationing cost involved in a power supply system in the framework of the standard inventory analysis, instead of the conventional marginalist approach of welfare economics, he has also formulated indirectly a peak period price in terms of rationing cost (Pillai 2002). The present paper is in continuation of these refinements.

3.3 Empirical Studies on Peak-Load Pricing

As already mentioned, theoretical interest on peak load pricing has waned over time and given way to empirical analysis of residential electricity demand by time of use. Most of the published studies have sought to estimate electricity demand by time-of-day, using data at the household level obtained from 'rate experiments'. During the last three decades, in countries such as the US (see, for example, Faruqui andMalko 1983 and Faruqui and George 2002), the UK (see Henley 1994) and France(see Aubin *et al*. 1995), several demonstration projects on residential electricityconsumption by time-of-use were promoted in an attempt to better understand the effects of time-of-day pricing on residential electricity consumption. Generally, in a rates experiment, residential consumers of an electric utility are selected randomly and placed on various time-of-use rates for a time horizon ranging from two to six months. The electric utilities collect monthly data on the electricity consumption of each of the selected customers during various daily time periods, which on aggregation provide a data set on residential time-of-use electricity consumption. Among the studies making use of such data set we have on the one hand those undertaken by Hill *et al*. (1983) and Filippini (1995a) that analyze the electricity demandby time-of-use using a system of log-linear demand equations in an 'ad hoc' way; that is, the models do not reflect completely the restrictions imposed by the neo-classical theory of consumer behaviour. On the other hand are studies by Caves *et al*. (1980), Aubin *et al*. (1995), Filippini (1995b), Baladi *et al*. (1998) that analyze the allocation of electricity expenditure to peak and off-peak consumption by using conditional demand system. For an overview of these studies see Hawdon (1992) and, recently, Lijesen (2007) and Faruquiand Sergici (2008); for a review on price and substitution elasticities under time-of-use rates, see Acton and Park (1984) and King and Chatterjee (2003).

Empirical evidences on the response of larger commercial and industrial customers to real time pricing (RTP) are reported in Patrick and Wolak (2001), Boisvert *et al*. (2007), Herriges *et al*. (1993), and Taylor *et al*. (2005). Barbose *et al*. (2004) provides acomprehensive overview on real time pricing programmes operated by US utilities. On the other hand, in spite of significant hourly variation in the wholesale market price, most of the US residential customers are charged a near-constant retail price for electricity. The first significant effort to introduce real time pricing, that is, hourly market-based electricity pricing to residential customers (called Energy Smart Pricing Plan) was developed by Chicago Community Energy Cooperative in association with Commonwealth Edison (ComEd) as a voluntary programme with 1500 households in Chicago in 2003. The four-year pilot Plan demonstrated the potential benefits of real-time electricity pricing on a limited basis. Its success paved the way for expanding real-time pricing to all households across the state of Illinois, starting in 2007. Allcott(2011) evaluates this first programme to expose residential consumers to hourly real-time pricing and finds that the enrolled households were statistically significantly price elastic and that consumers responded by conserving energy during peak hours, but remarkably did not increase average consumption during off-peak times.

4. Modelling Optimal Time-of-Day Pricing of Electricity

Programming and simulation models are regularly used to compare the techno-economic performance of different combinations of power plants and to evaluate the optimal schedule. However, they generally tend to be impotent in revealing the underlying principles of the optimal plant mix. To analyze this problem, the marginalist approach has been widely employed by electric utilities that rely on thermal sources of power.11 But systems depending primarily on hydroelectric power have not received that much extent of analysis.¹² The marginalist

^{11.} See, for example, the seminal work of Turvey (1968).

^{12.} This may be because, except Canada, most of the industrialized countries make little use of hydro-power. Bernard (1989) presents a marginalist analysis of the specific characteristics of limited hydro-power in a Ricardian framework in the context of Canada.

approach, however, is constricted in its scope of comprehension in that it usually reduces the operation of a multi-reservoir multi-plant system to that of an 'equivalent' single composite reservoir.

Equivalent composite representation of multi-reservoir systems is often used by engineers in evaluating optimal operation of hydroelectric systems.13 In the absence of a well-knit sophisticated planning model and of accessibility to solution techniques, and in view of intricate complications involved in dynamic analysis, such simple, static model comes in handy with the essential features to be analyzed for structuring long-run marginal cost (LRMC). Again it is an immediate alternative for taking into account the stochastic inflows, and it enables the use of stochastic dynamic programming.¹⁴

In what follows we present a simple, static model based, in general, on Turvey and Anderson (1977, Ch.15) and Munasinghe and Warford (1982; Ch. 4), but sufficiently modified to incorporate diverse technology, rationing costs and also soft deterministic equivalents of chance constraints representing stochastic demand and inflows. The model is solved for two types of power systems–pure hydro and hydrothermal under four structural assumptions–first-best, second-best, monopoly and constrained monopoly. The model analysis is followed by the derivation of a simple formula for outage costs, included in the objective function of the model.

4.1 Seasonal Cost Structure of A Hydro-Power System

The power generation of a hydro-system is subject to two constraints, viz., the available hydraulic energy (i.e., kinetic energy of falling water) that drives the turbines and the available installed capacity that sets a ceiling on the pace of conversion of hydraulic energy into electric energy. Given the capacity, hydraulic energy is determined

^{13.} See, for instance, Arvanitidis and Rosing (1970 a and b).

^{14.} See Neto, Pereira and Kelman (1984).

jointly by nature (rainfall) and by engineering works (dam, river diversion and dredging). The seasonality of water inflows entails storages for impounding water in the wet season to help meet the dry season requirements. Storage begins and rises with the wet season and once the reservoirs are full, spilling and/or sluicing occurs and continues as long as effective inflow exceeds energy demand. Discharge begins as the latter outgrows the former and consequently reservoir level falls. If the spilling and sluicing period spans quite long with a likelihood of this pattern recurring for many years, then the marginal costs of energy in the wet season will be essentially zero; because, with the energy inflows exceeding energy demands plus storage, extra energy in the wet season can be generated just by running through the turbines more water that might otherwise be spilled or sluiced away, provided there is enough plant capacity. The operating and maintenance (O&M) costs may increase a little to make up marginal costs.

In contrast, during the dry season, when energy inflows skimp in relation to outflows, extra reservoir capacity is required to meet extra energy demands and the corresponding costs of providing storage capacity represent marginal energy costs during the dry season. In certain instances allocating a fraction of the dam costs to the capacity costs may be justifiable, which, however, may depend on the nature of the specific case: for example, whether or not more storage is required to firm up the additional capacity.

Given this picture of supply cost characteristics, if we now superimpose on it demand for power with its random features bouncing between peak and off-peak points, we get an optimal schedule of generating costs.

Now the above model with the system assumptions can be more compactly and precisely be couched in terms of a marginalist approach. First we turn to the assumptions designing the load duration curve (LDC), pivotal to our analysis.

4.2 Load Duration Curve

Our models consider only independent demands during a period divided into two seasons, wet and dry, $s = w$, d. The time-varying demand for power during each season is represented by a LDC (Fig. 1) which describes the width of the time-interval, θ , that demand equals or exceeds a given capacity level *q*:

$$
q = G(\theta), 0 \le \theta \le T;
$$
\n
$$
\dots (4.1)
$$

where *T* is the total hours during the season. Because of its monotonicity and continuity, the function $G(\theta)$ can be inverted to obtain the width of the time-interval when capacity level q is in use:

$$
\theta = G^{-1}(q) \equiv \Gamma(q), 0 \le q \le \overline{q} = G(0) = \text{peak load.}
$$
 ... (4.2)

The LDC is broken down into two discrete blocks, *t*, of power demand – peak and off-peak, $t = p$, *o*.

Fig. 1: Load duration curve

4.3 The First-Best Prices

4.3.1 All-Hydro System

The first model considers the ramifications of the state-owned utility's welfare-commitments for its pricing policy. The mathematical formulation of the model portrays the maximization of the sum of consumers' and producers' surplus, given by the integrals of inverse demand curves less the costs:

$$
W = \sum_{s} \sum_{t} \int_{0}^{Q_{st}} P_{st}(Y_{st}) dY_{st} - \left\{ \sum_{i} \beta^{i} q^{i} + \sum_{i} \rho^{i} R^{i} + \sum_{i} \sum_{s} \sum_{t} b^{i} q^{i}_{st} \theta_{st} + \sum_{s} \sum_{s} \sum_{t} r_{s} Z_{st} \theta_{st} \right\}
$$
 ... (4.3)

where

- Q*st* : demand in season *s*, period *t*;
- *qi*: power capacity of the ith hydro-plant (kw);
- β^i : : the corresponding constant annuitized marginal (turbine) capacity cost;
- R^i . : peak reservoir capacity (hydraulic energy) of the *i*th plant (kwh)
- ρ^i : : the corresponding constant annuitized marginal capital cost;
- q^i_{st} : power output of the plant i in season s , period t (kw);
- h^i : the corresponding (output inelastic) constant marginal operation and maintenance costs.
- $\theta_{\rm st}$: the length of the period *t* in season *s*;
- Z_{gt}: size of power cut (i.e., excess of demand over power generated) in season *s*, period *t* (kw); and
- r_s : constant marginal penalty cost of energy demanded but not supplied because of capacity or energy shortage in season *s*.

This maximization is subject to a number of constraints. First let us consider what the French writers call the 'guarantee conditions', to ensure supply, to an acceptable probability limit, in the face of contingencies–water shortages in dry seasons, peak-load above mean expectations, or plant outages. These conditions are incorporated into the model in two forms: one for peak power supplies and the other for energy supplies in critical periods. Thus the first one gives the chance constraint that the capacity will be enough to meet the peak-load at least 100 per cent of the time:

 $Pr{\{\sum_i a^i \geq 0^*\}} \geq \alpha$

where Q^{*} is the stochastic peak load and $0 \leq \alpha \leq 1$.

This guarantee condition is often simplified in practice in terms of a 'margin of available capacity' over and above that required to meet the mean expected peak demand, as found by Cash and Scott (1967) while reviewing the practices in European countries in planning system reliability. Thus itis expressed as

 Σ _i $q^i \geq$ E (Q^*) (1 + PRM)

where PRM refers to percent reserve margin. This constraint may better be added implicitly to the model, since its effect is tantamount to interpreting Σq^i as actual capacity less an allowance for the risk of peakload outgrowing its mean expected value; that is, Σ*q*ⁱ is $1/(1 + PRM)$ of actual capacity which in turn implies that β^{i} s are now $(1 + PRM)$ times the cost of a kw of new capacity. Hence, hereafter β^i _s represent these adjusted costs and $qⁱ$ s, the available capacities.

The second guarantee condition, relating the energy availability especially in dry seasons, takes on the chance constraint that the total power output may be insufficient to meet the instantaneous demand at most $100(1 - \alpha_{st})$ per cent of the time:

$$
\Pr\left\{\Sigma_i q_{st}^i \geq Q_{st}, \right\} \geq \alpha_{st}, \ 0 \leq \alpha_{st} \leq 1 \tag{4.4}
$$

The inclusion of a penalty cost term in the objective function is in fact a direct effect of this chance constraint likely to be violated, i.e., the social cost of the failure to meet requirements. Hence suffice it to replace this constraint by the following relation:

$$
\Sigma_{\rm i} q^i_{\,\,st} + Z_{\rm st} - Q_{\rm st} = 0, \,\forall s, \, t; \qquad \qquad \text{(dual variables } \mu_{\rm st}) \quad \dots (4.4')
$$

an equality, Z_{st} being the shortfall.

Next are the capacity constraints that plant output can never exceed the corresponding available capacity:

$$
q^{i}_{st} - q^{i} \le 0, \ \forall \ i, \ s, \ t; \ \text{(dual variables } C^{i}_{st}) \tag{4.5}
$$

The stochastic water flows and storage are captured in a chance constraint that the energy release during a season plus water in storage at the end of the period cannot exceed, at least 100αs per cent of the time, the inflow during the period (corrected for evaporation and seepage) plus the water in store at the beginning:

$$
Pr{\Sigma_i q^i_{st} \Theta_{st} + S^i_{s-1} \leq \dot{I}^i_{s}} \geq \alpha_{s}, \qquad \qquad \dots (4.6)
$$

where S^i _s is the water in *i*th storage at the end of *s*, l^i _s is water inflow into it corrected for losses during *s*, and $\leq \alpha_s \leq 1$; all variables are expressed in kwh.

Conversion of this chance constraint into its equivalent deterministic form requires information on the probability distribution of the stochastic inflow I^i _s. Assuming the probability distribution is known and its fractiles are completely determined by its mean, $E(Iⁱ_s)$, and standard deviation, σI^i _s, and defining $k\alpha$ _s by the relationship $F(k\alpha_s) = \alpha_s$, $0 \le \alpha_s \le 1$; where $F(.)$ is the cumulative distribution function of $\{I^i_s - E(I^i_s)\}\sigma I^i_s$, the chance constraint may be written as

$$
\Sigma_{\mathbf{i}} q^i_{st} \Theta_{st} + S^i_{s-1} \leq \mathbf{E} \left(I^i_{s} \right) + k_{\alpha s} \sigma I^i_{s} \tag{4.6'}
$$

which is its deterministic equivalent.

For a marginalist analysis, however, this specification lends little help; and hence for practical purposes, we qualify the energy release,

 $qⁱ_{st} \theta_{st}$, in order to atone for the stochastic impacts of inflow, with a water availability factor, ω_s^j , which in effect, if lower, imposes a penalty in terms of higher storage costs. Thus the water balance constraint we consider is

$$
\sum_{t} \frac{q_{st}^{i}}{\omega_{s}^{i}} \theta_{st} + S_{s}^{i} - S_{s-1}^{i} \le \mathbb{E}(I_{s}^{i}), \quad \forall \ i, s \ (\text{dual variables } H_{s}^{i}). \quad \dots (4.6^{\circ})
$$

The last, upper storage constraint, requires that the quanta of water stored, S^i_s , can never exceed capacity, R^i :

$$
S'_{s} - R' \leq 0, \forall i, s \quad \text{(dual variables } X'_{s}). \tag{4.7}
$$

The Kuhn- Tucker conditions for maximization subject to these constraints are:

$$
Q_{sf} > 0; P_{sf} - \mu_{sf} = 0; \tag{4.8}
$$

$$
q \geq 0 \; ; \; -\beta^i - \sum_s \sum_t C_{st}^i = 0 \; ; \qquad \qquad \ldots \; (4.9)
$$

$$
S_s^i \geq 0; H_s^i - H_{s+1}^i + X_s^i \leq 0; \qquad \qquad \dots (4.10)
$$

$$
R^{i} > 0; -\rho^{i} - \sum_{s} X_{s}^{i} = 0; \qquad \qquad \dots \tag{4.11}
$$

$$
Z'_{s2} \geq 0 \; ; -r_s + \mu_{s1} \leq 0 \; ; \tag{4.12}
$$

$$
q_{st}^{i} \geq 0; \ -b^{i}\theta_{st} + C_{st}^{i} + \frac{H_{s}^{i}}{\omega_{s}^{i}}\theta_{st} + \mu_{st} \leq 0; \qquad \qquad \dots (4.13)
$$

The last equation when $qⁱ_{st}$ is positive yields seasonal time-ofuse long-run marginal cost per kwh, μ_{sf}/θ_{st} , and together with the first one gives the usual first-best solution, $P = MC$. Assuming there is only one hydro-plant in the system, an equivalent composite reservoir case, and S_s and q_{st} are positive, we get the following results.

The water constraint (4.6") is not binding during spilling periods, $s = w$, and hence H_w is zero, which is its lower value. From (4.13) we have, then, during the wet season

$$
\frac{\mu_{wt}}{\theta_{wt}} = b - \frac{c_{wt}}{\theta_{wt}}.
$$
\n(4.14)

The capacity constraint (4.5) is not binding during the off-peak period, $t = 0$, so that C_{wo} is zero. Thus marginal cost of hydro-generation during wet off-peak periods is just equal to *b*, the O & M costs per kwh involved.

When the capacity constraint is binding so that C_{wt} is positive in periods $t = p$, (4.9) gives $-C_{wp} = \beta$ and hence marginal cost per kwh during wet peak periods is

$$
\frac{\mu_{wp}}{\theta_{wp}} = b + \frac{\beta}{\theta_{wp}}.
$$
\n
$$
\dots(4.15)
$$

The upper storage constraint (4.7) may be binding for several successive periods of spilling; but X_w will be positive only for the last of these spilling periods because extra reservoir capacity is useful only if it provides more water for discharge. Hence, if $d + 1$ is the first draw down (discharge) period, then from (4.10) we get $H_d = X_{d-1}$, ('.' $H_{d-1} = 0$). As X_s is positive only in *d*-1, (4.11) gives $\rho = -X_{d-1}$, so that $-H_d = \rho$. Hence in the dry off-peak periods, marginal cost per kwh is

.... (4.16)

i.e., the unit 0 &M cost plus the annuitized cost per kwh of storage capacity weighted by the water availability factor. In contrast, in the dry peak period - $C_{dp} = \beta$ and hence

$$
\frac{\mu_{dp}}{\theta_{dp}} = b + \frac{\rho}{\omega_d} + \frac{\beta}{\theta_{dp}}.
$$
 (4.17)

4.3.2 Hydro-Thermal System

Now we will find out the rules for optimal plant mix and the corresponding prices when there are two plants in the system. This will be such as to be in keeping with the direction of our empirical exercise (in the next chapter), so that we assume that a thermal plant is added to our system with a single representative reservoir. Thermal plant will be used in the dry season continuously on base-load operation with hydro meeting the peak; and vice-versa in the wet season. Such a specification entails new definitions for some of the elements in our earlier model. Let us denote the sets of hydro and thermal plants by *h* and *f* respectively; then our generalized model (4.3) becomes

$$
W = \sum_{s} \sum_{t} \int_{0}^{Q_{st}} P_{st}(Y_{st}) dY_{st} - \left\{ \sum_{i \in h, f} \beta^{i} q^{i} + \sum_{i \in h} \rho^{i} R^{i} + \sum_{i \in h, f} \sum_{s} \sum_{t} b^{i} q^{i}_{st} \theta_{st} + \sum_{s} \sum_{s} \sum_{t} r_{s} Z_{st} \theta_{st} \right\}
$$

where b^i , $i \in h$, f , are now O & M costs for hydro plants and fuel costs plus O & M costs for thermal plants. It needs no mention that the water balance constraints apply only to the hydro-plants. Hence the last of the Kuhn-Tucker conditions may be rewritten more specifically as

$$
q_{st}^{i} \ge 0; \ -b^{i}\theta_{st} + C_{st}^{i} + \frac{\mu_{s}^{i}}{\omega_{s}^{i}}\theta_{st} + \mu_{st} \le 0; \quad i \in h; \tag{4.13'}
$$

and

$$
q_{st}^{i} \geq 0; -b^{i}\theta_{st} + C_{st}^{i} + \mu_{st} \leq 0; \quad i\in f; \tag{4.13'}
$$

Now let us consider the system with two plants, one hydro (*h*) and one thermal (*f*), in the dry season, assuming $q^i_{dt} > 0$, $i = h$, *f*. Then, eliminating μ_{dt} and substituting for C^i_{dt} , $i = h$, f , and for H^i_{dt} , $i = h$, in the above equations, we get the familiar rules for optimal load scheduling. To be specific,

$$
\frac{\beta^f - \beta^h}{\theta_{dt}} + b^f = b^h + \frac{\rho}{\omega_d}, \qquad \qquad \dots (4.18)
$$

i.e., the marginal generating cost should be equal at the optimum for both the plants. More precisely, it requires that the marginal capacity cost per kwh saved if hydro-plant were used instead of thermal, should be equal to the savings in marginal running cost per kwh if thermal were operated instead of hydro. It also implies that if the hydro-plant has cheaper marginal running cost, then it should be more expensive to construct.

Note that the R.H.S in (4.18) is the optimal price $(= MC)$ per kwh in the dry off-peak period for a single hydro-plant system. Hence on the strength of the economic rationale that extra thermal capacity means commensurately less hydro-capacity in need and therefore a saving in its cost, the L.H.S. in (4.18) may be taken as the marginal cost per kwh in the dry off-peak period for the hydro-thermal system¹⁵. And in the peak period, as we know, the MC per kwh will be higher by β^{h}/θ_{dn} , i.e.,

This may be rewritten as

$$
\frac{\mu_{dp}}{\theta_{dp}} = \beta^h \left(\frac{1}{\theta_{dp}} - \frac{1}{\theta_{do}} \right) + \beta^f \left(\frac{1}{\theta_{do}} \right) + b^f, \tag{4.19'}
$$

or,
$$
\mu_{dp} = \beta^h \left(1 - \frac{\theta_{dp}}{\theta_{do}} \right) + \beta^f \left(\frac{\theta_{dp}}{\theta_{do}} \right) + b^f \theta_{dp}.
$$
 ... (4.19")

In other words, peak-load operation of the hydro requires a capacity $1/\theta_{d0}$ less than its peak capacity, but no additional hydraulic power, the decrease being compensated for by the thermal with extra fuel provisions. That is, as (4.19) indicates, 16 it is possible for adding one kw of hydrocapacity to be used during θ_{dn} hours without extra hydraulic energy. Since hydraulic energy remains the same, this leaves θ_{dn}/θ_{do} of a hydro-plant without hydraulic energy during θ*do* hours, so that the net capacity increase is only $1-\theta_{dp}/\theta_{do}$ with no change in energy. To counter this deficiency, however, both capacity, $(\theta_{dp} / \theta_{do} \text{kw})$ and energy, $(\theta_{dp}$ kwh) provisions are required for the thermal.

Now it is straightforward to find out the marginal costs in the wet season, when hydro will be continuously on base-load operation and thermal on the peak. The same logic as above yields an off-peak price in

^{15.} See Turvey and Anderson (1977, Ch. 15).

^{16.} For a similar result for two hydro-power'sites', see Bernard (1989).

terms of i) cost savings if thermal were used instead of hydro, plus ii) O & M costs of hydro, (the sum to be equal to thermal fuel costs). The peak price is obtained by adding to it, the marginal annuitized thermal capacity costs per unit.

Below we tabulate the first-best seasonal time-of-day (SID) prices per kwh of electricity for an all-hydro (single representative reservoir) system and a hydro-thermal (one hydro-one thermal: both representative) system:

4.4 The Monopoly Prices

Our second model is set to look for the pricing implications of the utility's objective ingrained in its monopoly status to maximize profit rather than welfare. The relevant objective function is

$$
\Pi = \sum_{s} \sum_{t} P_{st} Q_{st} - \text{COST}, \tag{4.20}
$$

where Π denotes profit and COST refers to the cost terms in parentheses in (4.3) for a pure hydro system and in (4.3') for a hydro-thermal one. The maximization subject to the relevant production constraints we have considered earlier - (4.4) to (4.7) - yields the monopoly prices which we tabulate below for our two systems:

where e_{st} , $s = w$, d ; $t = o$, p ; is the price elasticity of demand in season *s*, period *t*.

As usual, monopoly price attaches an elasticity term to the welfare price and is hence pregnant with price discrimination potential. Depending upon the degree of the period elasticity and marginal capacity cost per kwh, there is a possibility of pricing reversals, as found by Bailey and White (1974).

4.5. The Ramsey Prices

Our constant cost model ensures under the marginal cost pricing rule that the utility just exactly breaks even. The guidelines laid down by the Venkataraman Committee characterize the Electricity Boards in effect as commercial-cum-service organizations and require them not merely to break-even, but also to generate a surplus after meeting all expenses properly chargeable to revenues, including O & M expenses, taxes, depreciation and interest.(Government of Kerala 1984: 33-34). Hence we add to the welfare function model an additional constraint of the following form:

 $\Pi \geq \Pi_0$ (dual variables γ), (4.21)

where Π is as in (4.20) and Π_0 is some desired profit level. The maximization of the welfare function [(4.3)/(4.3')] subject to the relevant production constraints, $(4.4) - (4.7)$, and the profit level constraint (4.21) gives the following second-best prices for our two simple systems.

Here the prices equal marginal costs inflated with weights imposed by the profit level constraint as well as the price-elasticity of period demand. These Ramsey prices warrant that the price-cost margin for each period is proportional to the marginal deficit (MR less MC) incurred in that period.17 The Bailey - White pricing reversal possibility appears here also.

4.6. Constrained Monopoly Prices

It needs no note that care should be taken to reduce the abuse of monopoly motive to push up the prices beyond certain levels and thus to safeguard the socio-economic development. At the same time the

^{17.} Cf. Baumol and Bradford (1970) and Boiteux (1949, 1956). Our profitensuring pricing rules are reminiscent of those in the general model of optimal departures from marginal cost pricing to deal with the deficit dilemma in the context of increasing returns in capacity provision.

utility should strive to reap a reasonable return on its capital. Hence on the assumption of a fair return, *s*, larger than the market cost of capital, *k*, the monopoly behaviour (4.20) may be constrained under a rate of return regulation of the form:

$$
\sum_{s} \sum_{t} P_{st} Q_{st} - \{ \sum_{i} \alpha \beta^{i} q^{i} + \sum_{i} \rho^{i} R^{i} + \sum_{i} \sum_{s} \sum_{t} b^{i} q_{st}^{i} \theta_{st} + \sum_{s} \sum_{t} r_{s} Z_{st} \theta_{st} \} \le 0,
$$
\n(dual variables λ); (4.22)

where $\alpha = s/k > 1$, and the superscript *i* should be defined in accordance with whether the system is pure hydro or hydro-thermal one (cf. Averch and Johnson (1962)).

Maximizing profit subject to the original set of constraints, $(4.4) - (4.7)$, and (4.22) , we get the following time-varying prices for our two systems under consideration:

First let us consider the hydro system; a surprise springs up in that the rate of return regulation appears not to affect the off-peak pricing policy of the utility, if cross-elasticity effects are zero, as the off-peak prices under rate of return regulation in both the seasons are identical to those obtained for a profit-maximizing monopoly. All the onus of regulation falls on the peak prices.

In the case of hydro-thermal system the off-peak prices also bear the burden, as they are expressed in terms of capacity cost savings. Except for pure hydro off-peak periods, regulation sets MR equal to something less than MC; and thus the period prices, except the hydro off-peak ones, under rate-of-return regulation are lower than those of an unconstrained profit maximizer.

Comparing the prices under these four models, it is clear that, as expected, the monopoly prices constitute the upper bound of the price domain and the first best prices form the floor except when a higher value of λ is imposed upon the regulated monopoly. Between these lie other model prices, given enough flexibility for the concerned constraint to exert itself upon the respective model. Thus a very high value of λ (low γ) tends to constrict the constraint driving prices to the minimum.

So far we have assumed zero power cut. When Z_{st} is positive, (4.12) gives the marginal penalty cost, r_s , $s = w$, d, of supply in-swerving from demand orbit. In the next section we derive a simple formula for this outage cost and proceed to tabulate the seasonal rationing price structure under the four model assumptions.

4.7 Outage Costs: Pricing under Uncertainty

For convenience we deal with a pricing period in terms of a season divided into different time blocks. The energy demand in a given period *t* is assumed to be a continuous function of price, P_t , and a measurable function of the outcome of a random event. Also it is assumed to be independent of other period demands and is represented in additive form as

Dt(Pt, Ut) = Q *t (Pt) + Ut,* (4.23)

where $\overline{O}_t(.)$ stands for mean demand in period t and U_t is a random variable with $E(U_t) = 0$.

The gross benefit of electricity consumption is denoted by $W_t(Q_t, U_t)$, assumed to be an increasing concave function of the energy demand Q_t . The willingness to pay of the consumers can then be represented by the derivative of W_t ...), which should be equal to energy price at a consumption level of D_t ...); i.e.,

$$
W'_{t} \{D_{t}(\ldots)\} = P_{t}
$$
 (4.24)

We retain the conventional capacity and energy costs and ordering of the n-technology model. In the face of outages and monsoon failure, it is the available capacity that is of more practical significance. Let the available capacity vector be denoted by $\bar{q} = (a_1q_1, \dots, a_nq_n)$, where ai is the availability of the *i*th plant of installed capacity *qi*. The total available capacity of plants 1, ... ,*i* is then given by

$$
Z_i = \sum_{k=1}^{i} \bar{q}_k.
$$
 (4.25)

Since the power supply cannot exceed the available capacity, the actual power output in any period is

$$
Q_t(P_t, U_t, Z_n) = \min\{D_t(..), Z_n\} \qquad \dots (4.26)
$$

That demand is stochastic portends supply shortages and entails rationing costs.18 For simplicity we assume a linear outage cost with a constant marginal outage cost of *r* per unit of energy.19

The model seeks to maximize the expected net social welfare *w* defined as

 $w =$ expected social welfare – capacity costs – expected energy costs – expected outage costs. That is,

$$
w = \sum_{t} \mathbb{E}\{W_t[D_t(\ldots)]\} - \sum_{i} \beta^{i} q^{i} - \sum_{i} \sum_{t} \theta_t b^{i} \mathbb{E}\{q_t^{i}[D_t(\ldots), \bar{q}]\} - \sum_{t} \theta_t r \mathbb{E}\{D_t(\ldots) - Z_n\}.
$$
\n(4.27)

From the conventional concepts of ordering and total costs, it follows that plant *i* will be used in period *t* precisely when

$$
Z_{i-1} - \bar{Q}_t(.) \le U_t \le Z_i - \bar{Q}_t(.), \tag{4.28}
$$

so that

$$
E\{q_t^i[D_t(.), \overline{q}]\} = \int_{z_{i-1} - \overline{Q}_t(.)}^{z_i - \overline{Q}_t(.)} \{D_t(..) - Z_{i-1}\} dF_t(U_t), \qquad (4.29)
$$

where F_t is the cumulative distribution function (CDF) of U_t . Hence,

$$
\frac{\partial}{\partial q^i} \{-\sum_i \sum_t \theta_t b^i E[q_t^i(D_t(.),\bar{q})]\} = -\sum_t \theta_t b^i a^i \{1 - F_t[Z_i - \bar{Q}_t(.)]\}
$$

^{18.} For a detailed discussion on rationing costs, see Crew and Kleindorfer (1986, Ch.4,6).

^{19.} See Turvey and Anderson (1977, Ch. 14).

+
$$
\sum_{j=i+1}^{n} \sum_{t} \theta_t b^j a^j \{F_t [Z_j - \bar{Q}_t(.)] - F_t [Z_{j-1} - \bar{Q}_t(.)]\}
$$
. (4.30)

 $Thus²⁰$

Note that

$$
F_t[Z_i - \bar{Q}_t(.)] = \Pr\{U_t \le [Z_i - \bar{Q}_t(.)]\} = \Pr\{[\bar{Q}_t(.) + U_t] \le Z_j\}, \dots (4.32)
$$

i.e., the probability that demand does not exceed the available capacity of the first *j* types, *Zj*.

Rewriting (4.31) for $i = n$, we get

$$
\Pr\left\{ [\bar{Q}_t(P_t) + U_t] \ge Z_n \right\} = \frac{\beta^n}{\sum_t \theta_t a^n (r - b^n)}.
$$
\n(4.33)

which gives an explicit relationship between outage costs and optimal reliability criterion. The LHS of (4.33) is by definition, the loss of load probability (LOLP),the probability or the expected fraction of time that demand exceeds the available supply. From (4.33) we get

$$
r = \frac{\beta^n}{a^n \text{LOLP} \Sigma_t \theta_t}, \qquad \qquad \dots (4.34)
$$

the desired explicit expression for outage costs. It is evident that the higher the reliability level insisted upon, the higher the outage costs to stand.

Now we tabulate below the seasonal rationing prices yielded by (4.8), (4.12) and (4.34) for the two power systems under the four structural assumptions (with the additional constraints).1

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^{20.} Cf. Chao (1983); (4.34) is an adaptation of his result.

Let
$$
LOLPa_s^i \sum_t \theta_{st} = L_s^i
$$
; $i = h, f$; $s = w, d$.

i) Pure Hydro

ii) Monopoly Model

iii) Ramsey Model

iv) Constrained Monopoly Model

Note that the outage costs are with respect to the peak- load units and that the elasticity terms are the peak period ones.

Conclusion

It has long been advocated that the sale of electricity and other services, in which periodic variations in demand are jointly met by a common plant of fixed capacity, should be at time-differential tariffs. Despite a very rich tradition of modeling, theoretical refinements in peak load pricing have not attracted much attention of late. The present study has sought to model seasonal time-of-day pricing rules for electricity for two types of power systems - pure hydro and hydro-thermal in normal and exigent conditions under the various umbrellas of assumptions in the first-best, second-best, monopoly and constrained monopoly domains. These simple, static rules appear to be well-adapted for less developed power systems, and in the face of inaccessibility of computerized dynamic models, capable of being applied to actual tariff estimation.

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